MATHEMATICS 7

Ratio and Proportion

Distance Learning

Alberta EDUCATION

Module 3

Mathematics 7

Module 3

RATIO AND PROPORTION

Alberta
EDUCATION

Mathematics 7
Student Module Booklet
Module 3
Ratio and Proportion
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This document is intended for	
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Teachers (Mathematics 7)	✓
Administrators	
Parents	
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Other	

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Welcome

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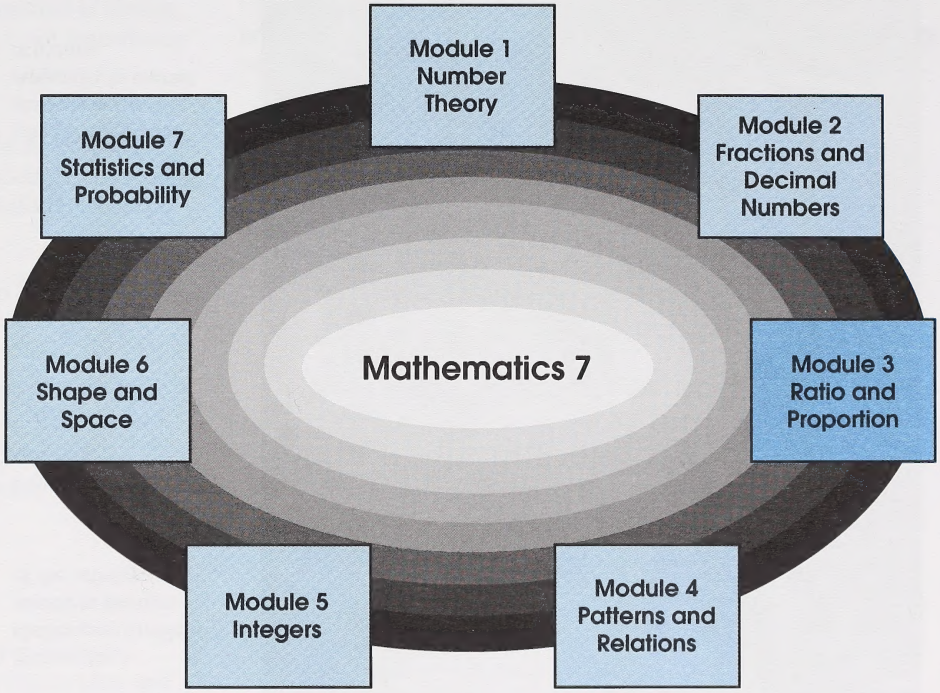
Welcome to Module 3. We hope you'll enjoy your study of Ratio and Proportion.

Mathematics 7 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

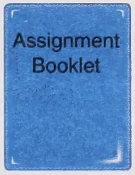
- Use computer software.

- Use a scientific calculator.

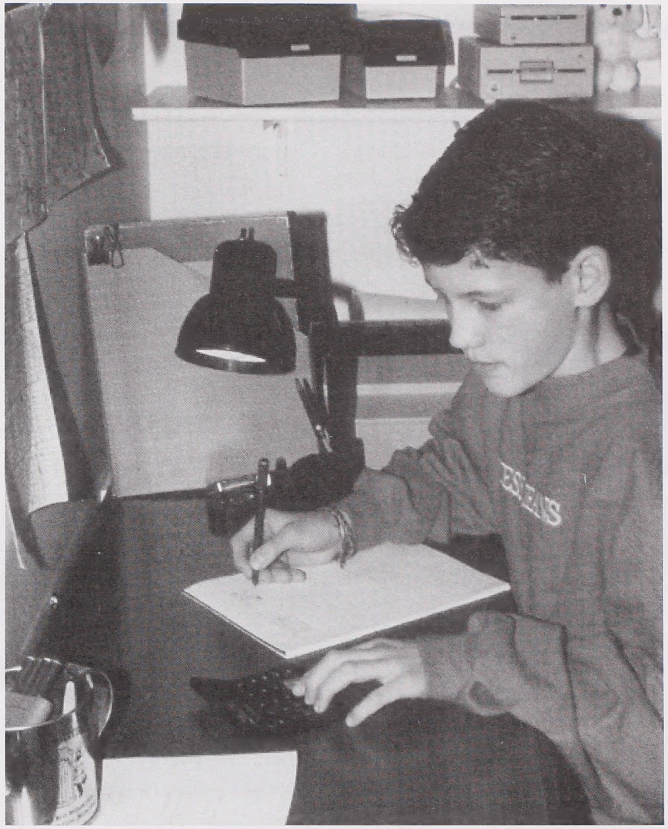
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The  icon is a cue that the problem will be related to the topic of the activity.

The  icon is a cue that the problem will provide a change of topic.

The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

Note: The Appendix in Module 1 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Appendix in Module 1 and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

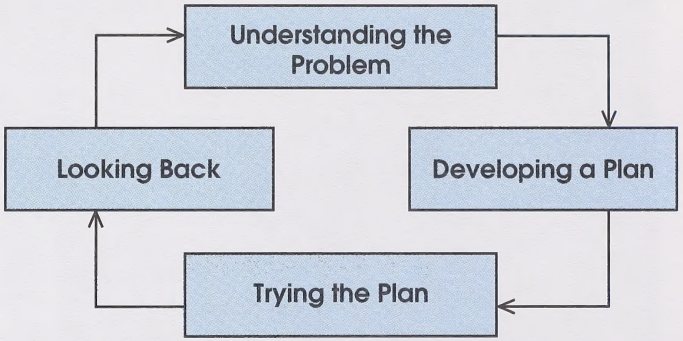
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

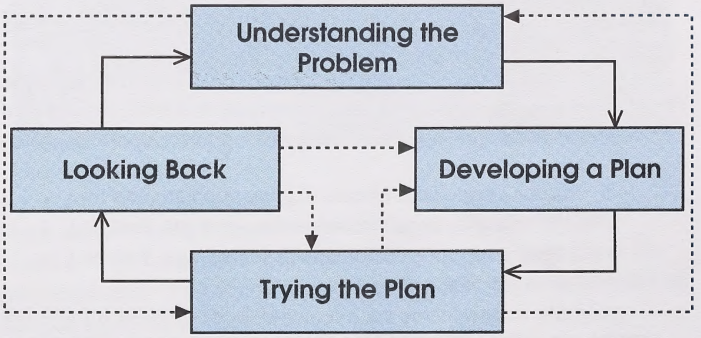
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



CONTENTS

Module Overview

Do you like bubble gum? Have you ever bought bubble gum from a machine like the one in the diagram? Many questions involving ratios, rates, and percents can be made up about bubble gum.

- What is the ratio of the number of red bubble-gum balls to the number of blue bubble-gum balls in a bubble-gum machine?
- If you can purchase five bubble-gum balls for $1, what is the cost per bubble-gum ball?
- If the bubble-gum machine is 50% full, what fraction of the machine is full?

You will solve problems like these in this module.

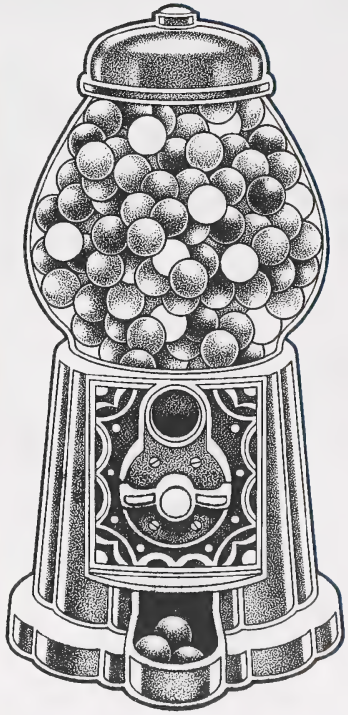
You will distinguish between a ratio and a rate. You will write ratios and rates in simplest form and find proportional ratios and rates. You will write ratios as percents. You will express percents less than 100% as fractions and decimal numbers. You will express percents greater than 100% as whole numbers, mixed numbers, and decimals. You will use proportions to solve ratio, rate, and percent problems.

Module 3
Ratio and Proportion

Section 1
Ratios and Rates

Section 2
Percents and Other Ratio Forms

Section 3
Proportions

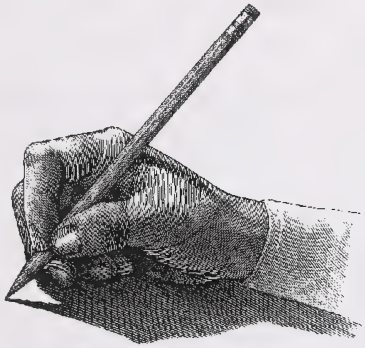


Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete four assignments. The mark distribution is as follows:

Section 1 Assignment	**25 marks**
Section 2 Assignment	**25 marks**
Section 3 Assignment	**25 marks**
Final Module Assignment	**25 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

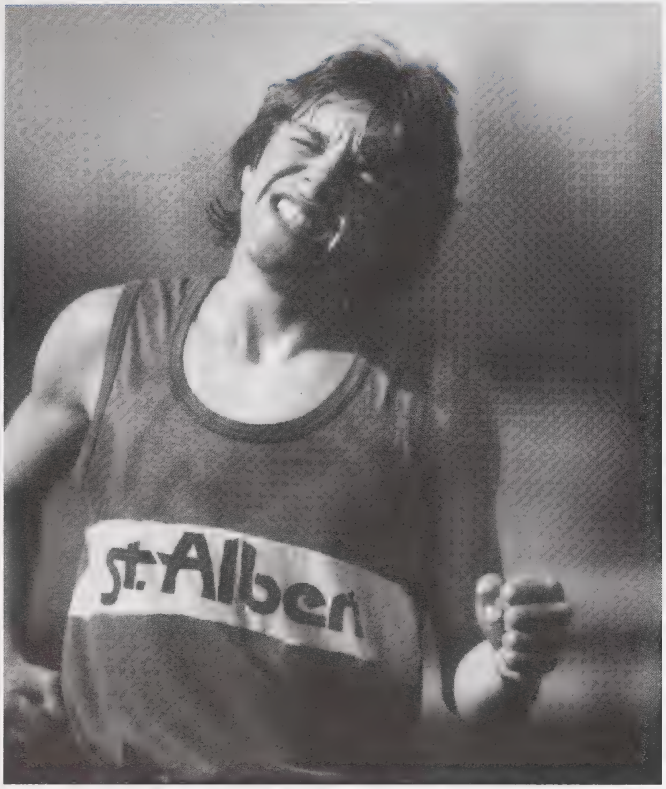


If you are working on a CML terminal, you will have a module test as well as a module assignment.

Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Section 1: Ratios and Rates



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Your heart is an amazing organ. Here are some facts about the heart.

- A woman's heart has a mass of about 260 g. A man's heart has a mass of about 312 g. So, the ratio of the mass of a woman's heart to the mass of a man's heart is about 5 to 6.
- When the body is at rest, the number of heartbeats in one minute is known as the resting rate. For women, the average resting rate is 75 beats per minute. For men, the average resting rate is 70 beats per minute.
- Physical activity strengthens the heart and allows it to work more efficiently. So, the resting rate of a trained runner may be as low as 40 beats per minute while the resting rate of an unfit adult may be as high as 90 beats per minute.

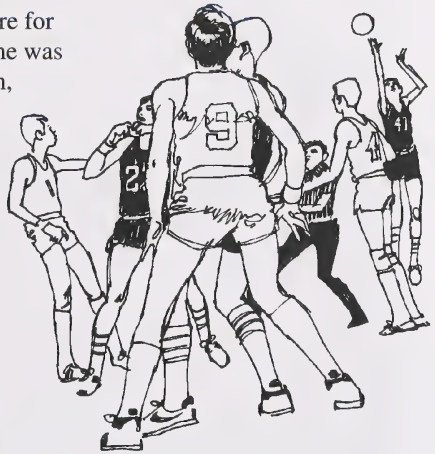
These facts regarding the heart use ratios and rates.

In this section you will distinguish between a ratio and a rate. You will also write ratios and rates in simplest form and find proportional ratios and rates.

Activity 1: Ratios

Comparing the Number of People, Animals, or Things

When you say that the score for yesterday's basketball game was 97 to 58 for the home team, you are comparing the number of points each team scored. You can express this relationship as a **ratio**.



The ratio of the number of points scored yesterday by the home team to the number of points scored by the visiting team was 97 to 58.

A ratio is a comparison that describes a number relationship.

When you write a ratio, be careful to indicate what is being compared. Also be careful of the order in which the numbers are given.

Example

There are a total of 8 fish in an aquarium. Of the 8 fish, 3 are striped and 5 are plain. Several ratios can be written to describe this number relationship.

- The ratio of the number of striped fish to the number of plain fish is 3 to 5.
- The ratio of the number of striped fish to the total number of fish is 3 to 8.
- The ratio of the number of plain fish to the number of striped fish is 5 to 3.
- The ratio of the number of plain fish to the total number of fish is 5 to 8.

Before answering questions 1 and 2 you may wish to gather the pattern blocks you used in Module 2 and arrange the blocks as shown in Figure 1 and Figure 2. The figures are made up of three kinds of pattern blocks—hexagons, parallelograms, and triangles.

Figure 1

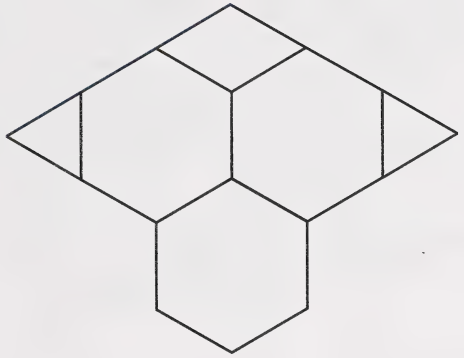
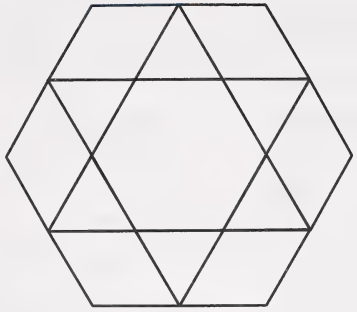


Figure 2



1. Complete each of the following statements comparing the different kinds of blocks in the two figures.

   a. The ratio of the number of parallelograms in Figure 1 to the number of parallelograms in Figure 2 is ____ to ____.

   b. The ratio of the number of hexagons in Figure 1 to the number of hexagons in Figure 2 is ____ to ____.

   c. The ratio of the number of hexagons in Figure 1 to the total number of hexagons in both figures is ____ to ____.

2. Use each of the following phrases to write ratios comparing the different kinds of blocks in Figure 1.

   a. 1 to 3 **b.** 3 to 2 **c.** 1 to 6

Check your answers by turning to the Appendix.

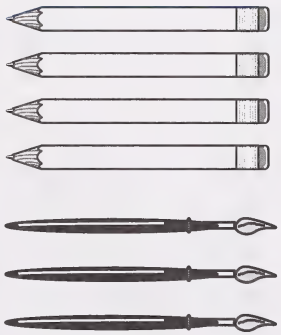
Different Forms

In Module 2, you discovered that fractions can be written in different forms. Proper fractions can be written as decimal numbers. Improper fractions can be written as mixed numbers or decimal numbers.

Ratios can also be written in different forms. Three forms of ratios are shown in the following example.

Example

There are 4 pencils and 3 brushes on the desk. Write the ratio of the number of pencils to the number of brushes.



Solution

Method 1: Using the word *to*

The ratio of the number of pencils to the number of brushes is 4 to 3.

Method 2: Using a colon

The ratio of the number of pencils to the number of brushes is 4 : 3.

This is read as "4 to 3."

Method 3: Using the fraction form

The ratio of the number of pencils to the number of brushes is $\frac{4}{3}$.

This is read as "4 to 3."

The first number in a ratio is called the **first term**. The second number in a ratio is called the **second term**.

For questions 3 and 4, express each ratio as a statement. Write each ratio three ways: using the word *to*, using a colon, and using the fraction form.

3. There are 5 soccer balls, 3 baseballs, 2 footballs, and 1 basketball in a box of sporting equipment.

a. What is the ratio of the number of soccer balls to the number of footballs?

b. What is the ratio of the number of basketballs to the total number of balls?

4. The Hawks hockey team won 41 games, lost 10 games, and tied 8 games.

a. Write the win-loss ratio.

b. Write the ratio of the number of wins to the total number of games played.

5. The boy-girl ratio in a club is 13 to 15.

a. What is the first term of this ratio?
b. To what does the first term in this ratio refer?

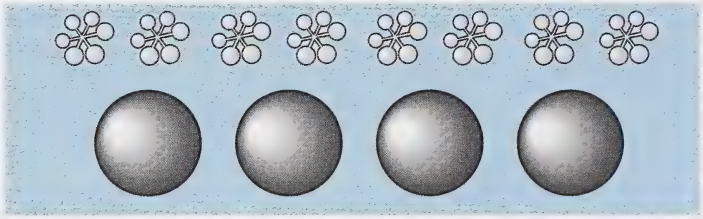
Check your answers by turning to the Appendix.

Lowest Terms

Sometimes it is desirable to simplify a ratio by writing it in **lowest terms** or **simplest form**.

A ratio is in simplest form if it is written with the lowest whole-number terms possible.

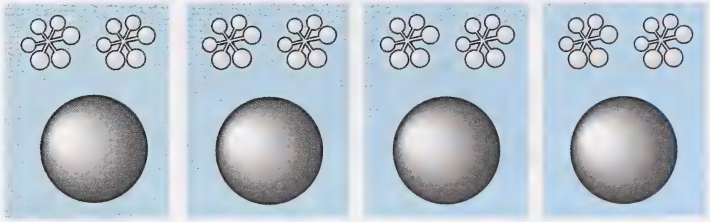
Example 1



The ratio of the number of jacks to the number of balls is 8 to 4. Write the ratio in lowest terms.

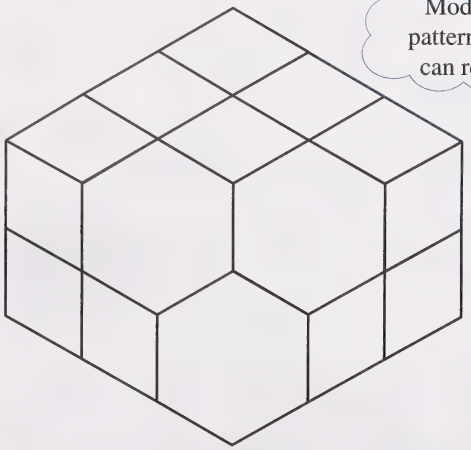
Solution

Regroup the jacks and balls so that the groups are equal, and there are the fewest possible number of jacks and balls in each group.



As the diagram shows, there are 2 jacks for every 1 ball. So, the ratio of the number of jacks to the number of balls is 2 to 1.

Use the following design, which was made with two different kinds of pattern blocks, for question 6. Express the ratios in question 6 in lowest terms.



Model the design with pattern blocks so that you can regroup the blocks.

6. a. The ratio of the number of hexagons to the number of parallelograms in the design is ____ to ____, or ____ to ____.

b. The ratio of the number of parallelograms to the total number of blocks in the design is ____ to ____, or ____ to ____.

Check your answers by turning to the Appendix.

In question 6, you used different groupings to write the ratio of the blocks in simplest form. You can also use division to write ratios in simplest form.

Example 2

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What is the ratio of the number of cans to the number of bottles in the photograph? Express the ratio in lowest terms.

Solution

In the photograph, there are 3 cans and 3 bottles.

$$\frac{\text{number of cans}}{\text{number of bottles}} = \frac{3}{3} = \frac{1}{1}$$

$\div 3$ above the fraction, $\div 3$ below the fraction

So, the ratio of the number of cans to the number of bottles in the photograph is 1 to 1.

Note: Another way to express the ratio in Example 2 is to say that there is 1 can for every 1 bottle, 1 can for each bottle, or 1 can per bottle.

You can think of a ratio as a rule. When you write a ratio in lowest terms, you are simplifying the rule. However, you are not changing the relationship of the numbers being compared.

Write each of the ratios in questions 7 and 8 as a statement. Express the ratio in lowest terms. Use the colon form.

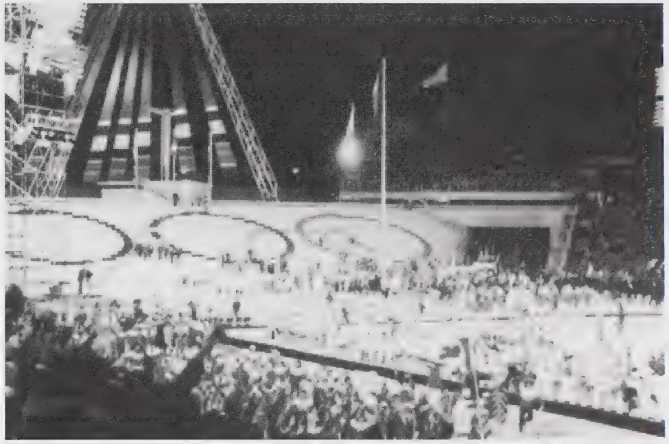
7. A pet store has 28 cats, 16 dogs, 10 rabbits, 20 birds, and 26 fish.

a. What is the ratio of the number of cats to the number of dogs?

b. What is the ratio of the number of rabbits to the number of birds?

c. What is the ratio of the number of fish to the total number of animals?

8. At the 1988 Winter Olympics in Calgary, West Germany won 2 gold medals, 4 silver medals, and 2 bronze medals. Switzerland won 5 gold medals, 5 silver medals, and 5 bronze medals.



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a. What is the ratio of the number of gold medals won by West Germany to the total number of medals won by West Germany?

b. What is the ratio of the number of gold medals won by Switzerland to the total number of medals won by Switzerland?

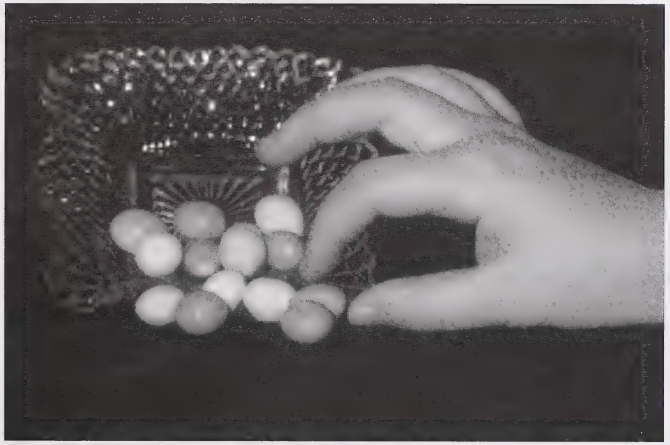
Check your answers by turning to the Appendix.

Proportional Ratios

If you think of a ratio as a rule, it is possible to write many **proportional ratios**.

Proportional ratios are equivalent ratios.

Example 1



As a child, when you shared some candies with your friend, you probably used the rule of one candy for your friend and one for you. So, the ratio of the number of candies your friend received to the number of candies you kept was 1 to 1.

However, you probably didn't hand out all the candies one at a time.

- You may have given your friend 2 candies and kept 2 for yourself.

$$\frac{\text{number of candies for your friend}}{\text{number of candies for you}} \qquad \frac{1}{1} \overset{\times 2}{=} \frac{2}{2}$$

The ratio of the number of candies you gave your friend to the number you kept was 2 to 2.

- You may have given your friend 3 candies and kept 3 for yourself.

$$\frac{\text{number of candies for your friend}}{\text{number of candies for you}} \qquad \frac{1}{1} \overset{\times 3}{=} \frac{3}{3}$$

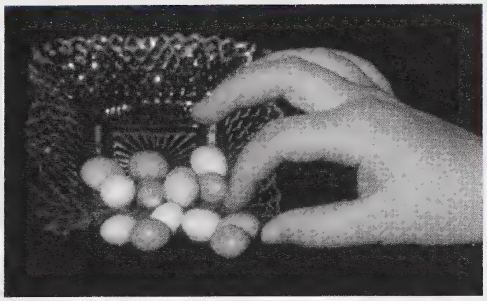
The ratio of the number of candies you gave your friend to the number of candies you kept was 3 to 3.

Note: The ratios in Example 1 are proportional ratios. The equations $\frac{1}{1} = \frac{2}{2}$ and $\frac{1}{1} = \frac{3}{3}$ are **proportions**.

A proportion is an equation that shows two equivalent ratios.

Example 2

As a child, when you shared some candies with a friend, you may have handed out the candies using this rule: Give your friend 2 candies for every 3 candies you keep.



You probably wouldn't have handed out the candies one at a time.

- You may have given your friend 4 candies and kept 6 for yourself.

$$\frac{\text{number of candies for your friend}}{\text{number of candies for you}} \quad \frac{2}{3} \overset{\times 2}{\underset{\times 2}{=}} \frac{4}{6}$$

The ratio of the number of candies you gave to your friend to the number of candies you kept was 4 to 6.

- You may have given your friend 6 candies and kept 9 for yourself.

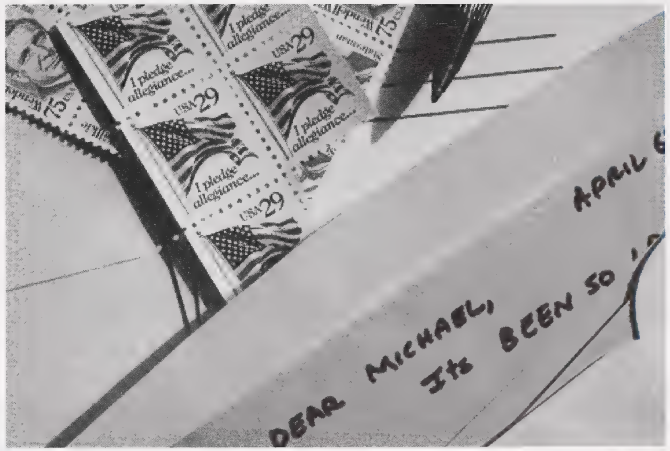
$$\frac{\text{number of candies for your friend}}{\text{number of candies for you}} \quad \frac{2}{3} \overset{\times 3}{\underset{\times 3}{=}} \frac{6}{9}$$

The ratio of the number of candies you gave to your friend to the number of candies you kept was 6 to 9.

Note: The ratios in Example 2 are proportional ratios. The equations $\frac{2}{3} = \frac{4}{6}$ and $\frac{2}{3} = \frac{6}{9}$ are proportions.

Write each of the ratios in questions 9 and 10 as a statement. Use the fraction form.

9. Johann buys 10 Canadian stamps for every 3 American stamps. Write three proportional ratios to describe the situation.



10. Mikka finds defects in 3 light bulbs out of every 100 she inspects. Write three proportional ratios to describe the situation.

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following questions.

11. **a.** Draw a set of 3 shapes for which the ratio of the number of circles to the number of triangles is 1 to 2.

b. Draw a set with more than 3 shapes, but fewer than 9 shapes, for which the ratio of the number of circles to the number of triangles is 1 to 2.

c. Draw a set with 9 shapes for which the ratio of the number of circles to the number of triangles is 1 to 2.

d. Draw a set with more than 9 shapes, but fewer than 15 shapes, for which the ratio of the number of circles to the number of triangles is 1 to 2.

Check your answers by turning to the Appendix.

Use a problem-solving strategy to answer the following question.

12. There are three animals in a row.

- One of the animals is a bull who always tells the truth.

- Another animal is a bear who sometimes tells the truth.
- Another is a rooster who never tells the truth.

The first animal says, "The bull is next to me." The second animal says, "I am the bear." The third animal says, "The rooster is next to me."

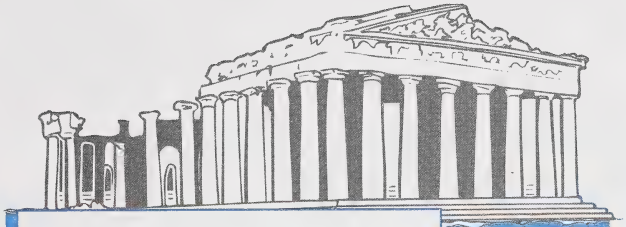
Name the animals in order.

Check your answers by turning to the Appendix.

Word Search Puzzle

Names of several ancient mathematicians are hidden in the following puzzle. The names may be read forward, backward, diagonally, up, or down. See how many names you can find from the given list.

Ancient Mathematicians



A	R	C	H	I	M	E	D	E	S	H	M
R	P	S	U	X	O	D	U	E	E	S	A
I	T	O	T	A	L	P	L	R	H	U	E
S	M	A	L	T	I	A	O	C	D	T	I
T	A	N	S	L	H	N	C	A	I	N	N
A	R	I	S	T	O	T	L	E	O	A	A
R	C	T	H	E	O	N	E	Z	C	H	I
C	O	N	O	N	O	U	I	N	L	P	T
H	T	O	N	D	I	L	C	U	E	O	A
U	E	M	E	N	E	L	A	U	S	I	P
S	A	R	O	G	A	H	T	Y	P	D	Y
P	A	P	P	U	S	A	I	P	P	I	H

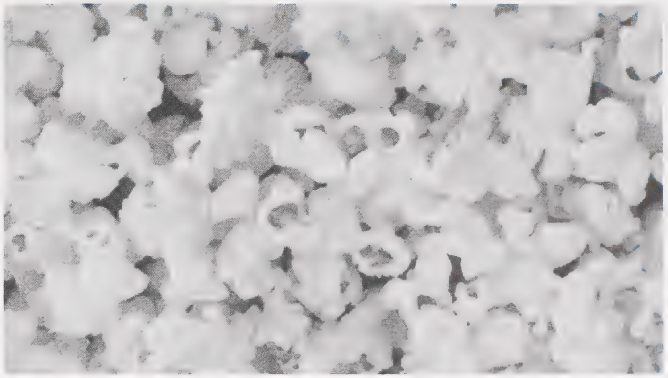
APOLLONIUS
ARCHIMEDES
ARISTARCHUS
ARISTOTLE
CONON
DIOCLES
DIOPHANTUS
EUCLID
EUDOXUS
HERON
HIPPIAS
HYPATIA
MENELAUS
PAPPUS
PLATO
PYTHAGORAS
THALES
THEON
ZENO

In this activity you wrote ratios to compare the number of people, animals, or things in two groups.

In the next activity you will write ratios to compare two quantities measured in the same units or in units that can be converted to the same unit.

Activity 2: More Ratios

Comparing Quantities with the Same Units

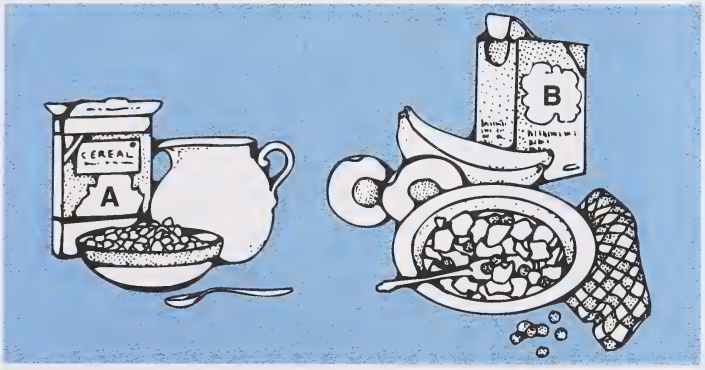


Did you know that the ratio of dried corn kernels to popped corn is 1 to 24? That is, if you pop 125 mL of dried corn kernels you will get 3000 mL of popped corn.

Ratios can be used to compare two quantities expressed in the same unit or in units that can be converted to the same unit.

The unit of measurement may be a standard unit such as a metre, gram, litre, or hour. It may also be a nonstandard unit such as a scoop or a pace. The unit is unimportant as long as the units of the quantities being compared are the same.

Example 1



There are 2 scoops of raisins in a box of Brand A cereal. There is 1 scoop of raisins in a box of Brand B cereal. Different ratios can be used to describe the situation.

- The ratio of the amount of raisins in Brand A to the amount of raisins in Brand B is 2 to 1.
- The ratio of the amount of raisins in Brand B to the amount of raisins in Brand A is 1 to 2.

Example 2

Maria made 5 L of punch by combining 3 L of cranberry juice and 2 L of ginger ale. Different ratios can be used to describe the situation.

- The ratio of the amount of cranberry juice to the amount of ginger ale is 3 to 2.
- The ratio of the amount of cranberry juice to the total amount of punch is 3 to 5.
- The ratio of the amount of ginger ale to the amount of cranberry juice is 2 to 3.
- The ratio of the amount of ginger ale to the total amount of punch is 2 to 5.

1. Franz is the tallest person in his family. His height is 199 cm. His father's height is 175 cm and his mother's height is 167 cm.

a. What is the ratio of Franz's height to his father's height?
b. What is the ratio of Franz's height to his mother's height?

2. Mavis is 13 years old and Emma is 12 years old.

a. What is the ratio of Emma's age to Mavis's age?
b. What is the ratio of Mavis's age to Emma's age?

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Check your answers by turning to the Appendix.

Different Forms

Ratios comparing quantities with the same units can be written in different forms.

Example

Jason and Ruth were painting a fence. Jason painted for 3 h and Ruth painted for 4 h. What is the ratio of the time Ruth spent painting to the time Jason spent painting?

Solution

Method 1: Using the word *to*

The ratio of the time Ruth spent painting to the time Jason spent painting is 4 to 3.

Method 2: Using a colon

The ratio of the time Ruth spent painting to the time Jason spent painting is $4:3$.

This is read as "4 to 3."

Method 3: Using the fraction form

The ratio of the time Ruth spent painting to the time Jason spent painting is $\frac{4}{3}$.

This is read as "4 to 3."

Notice that the units are not included in any of these ratio forms.

3. Jana made a shade of orange by mixing 2 L of yellow paint and 1 L of red paint. Write the ratio of the amount of red paint to the amount of yellow paint in each of the following forms.

   a. using the word *to*
   b. using a colon
   c. using the fraction form

4. A 10 m tree cast a shadow that was 7 m long. Write the ratio of the height of the tree to the length of its shadow in each of the following forms.

   a. using the word *to*
   b. using a colon
   c. using the fraction form

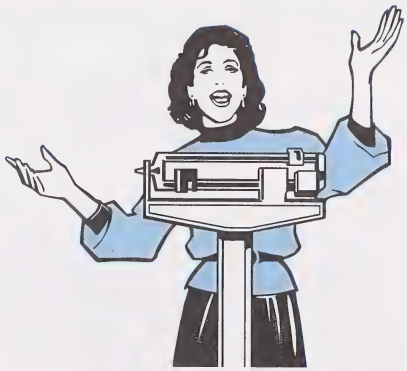
Check your answers by turning to the Appendix.

Lowest Terms

Sometimes it is desirable to write a ratio in simplest form. As you will recall, a ratio in simplest form has the lowest whole-number terms possible.

Example 1

Maria has a mass of 45 kg. Jean's mass is 54 kg. What is the ratio of Maria's mass to Jean's mass? Express the ratio in lowest terms.



Solution

$$\frac{\text{Maria's mass (kg)}}{\text{Jean's mass (kg)}}$$

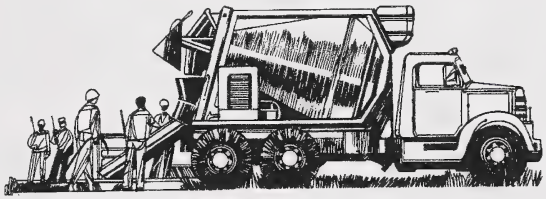
$$\frac{45}{54} \overset{\div 9}{=} \frac{5}{6}$$

The ratio of Maria's mass to Jean's mass is 5 to 6.

Express each of the ratios in questions 5 to 9 in lowest terms.

5. One tube of toothpaste contains 200 mL. A smaller tube of toothpaste contains 100 mL. What is the ratio of the amount of toothpaste in the larger tube to the amount of toothpaste in the smaller tube?

6. To make a concrete foundation, 300 kg of gravel are mixed with 450 kg of sand and 100 kg of cement.

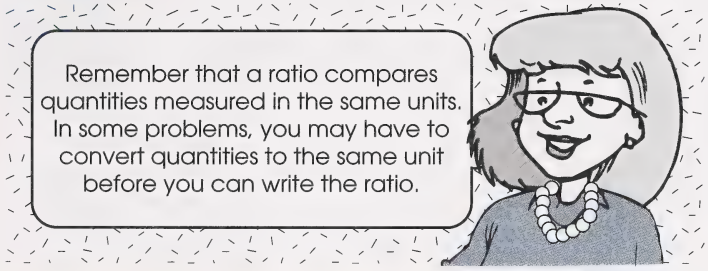


a. What is the ratio of the mass of gravel to the mass of sand?

b. What is the ratio of the mass of gravel to the mass of cement?

7. Brian slept 10 h on Monday night and 8 h on Tuesday night. What is the ratio of the time he slept on Monday night to the time he slept on Tuesday night?

Check your answers by turning to the Appendix.



Example 2

The diagram shows the jaw of a shark. The length of the shark's longest tooth is 3 cm. The length of the shark's body is 3.3 m. What is the ratio of the length of the shark's longest tooth to the length of the shark's body?

Solution

Step 1: Write the measurements in the same unit.

3.3 m = 330 cm

Step 2: Write the ratio in lowest terms.

$$\frac{\text{length of longest tooth (cm)}}{\text{length of body (cm)}} = \frac{3}{330} \overset{\div 3}{=} \frac{1}{110}$$

The ratio of the length of the shark's longest tooth to the length of the shark's body is 1 to 110.

8. Harvey has a pumpkin and a zucchini. The pumpkin has a mass of 2 kg. The zucchini has a mass of 250 g. What is the ratio of the mass of the pumpkin to the mass of the zucchini?

9. The body of an adult contains about 5 L of blood. A blood donor gives 450 mL of blood. What is the ratio of the amount of donated blood to the total amount of blood in the body?

Check your answers by turning to the Appendix.

Proportional Ratios

If you think of a ratio as a rule, it is possible to write many proportional or equivalent ratios.

Example

Suzanne and Peter invest in a business. Suzanne invests $5 for every $4 Peter invests.

An infinite number of ratios can be written using this rule.

- Suzanne could invest $500 and Peter could invest $400.

$$\frac{\text{Suzanne's investment (\$)}}{\text{Peter's investment (\$)}} \qquad \frac{5}{4} \overset{\times 100}{=} \frac{500}{400}$$

So, the ratio of Suzanne's investment to Peter's investment would be 500 to 400.

- Suzanne could invest $10 000 and Peter could invest $8000.

$$\frac{\text{Suzanne's investment (\$)}}{\text{Peter's investment (\$)}} \qquad \frac{5}{4} \overset{\times 2000}{=} \frac{10\,000}{8000}$$

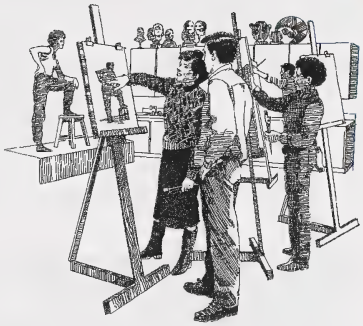
So, the ratio of Suzanne's investment to Peter's investment would be 10 000 to 8000.

Note: In the example, the ratios are proportional ratios. The equations $\frac{5}{4} = \frac{500}{400}$ and $\frac{5}{4} = \frac{10\,000}{8\,000}$ are proportions.

In questions 10 and 11 write statements and express the ratios in fraction form.

10. When you make orange juice from cans of concentrate, the rule is 1 can of concentrate to 3 cans of water. Write three proportional ratios to describe the situation.

11. It has been recommended that the optimum number of students to the number of teachers is 20 to 1. Write equivalent ratios to describe this situation.



Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

12. Beth has only a 5-L pail and a 3-L pail. Neither of the pails has any marking on it.

a. Explain how Beth can measure exactly 2 L.

b. Explain how Beth can measure exactly 1 L.

c. Explain how Beth can measure exactly 4 L.

Check your answers by turning to the Appendix.

Did You Know?

Mathematicians Making Things[1]

Mathematicians have made some very unusual things. For example, Archimedes (287–212 B.C.) made some huge hooks to lift enemy ships out of the water. He also made some long poles that could be used to throw rocks at an enemy.

In Activity 2 you wrote ratios to compare two quantities measured in the same unit or in units that can be converted to the same unit.

In the next activity, you will compare two quantities measured in different units. You will also compare the number of people, animals, or things in a group to a quantity.

[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*, by Merle Mitchell. Copyright 1978 by the National Council of Teachers of Mathematics.

Activity 3: Rates

Comparing the Number of People, Animals, or Things with a Quantity

The most densely populated region in the world is Macau on the southern coast of China. It has an estimated population of 367 000 in an area of 18 km^2.

A **rate** can be used to compare the number of people in Macau to the area of Macau.

A rate is a comparison that describes a unit relationship as well as a number relationship.

Rates can be written in different ways.

Example

When you say that a person can type 700 words in 9 min, you are comparing the number of words typed to the amount of time required. Write the typing speed as a rate.

Solution

Method 1: Using the word *per*

The typist typed 700 words per 9 minutes.

Method 2: Using a colon

The typist typed 700 words : 9 minutes.

This is read as "700 words per 9 minutes."

Method 3: Using the fraction form

The typist typed $\frac{700 \text{ words}}{9 \text{ min}}$.

This is read as "700 words per 9 minutes."

Method 4: Using a slash (solidus)

solidus

The typist typed 700 words/9 minutes.

This is read as "700 words per 9 minutes."

Note: Because the number of words typed is being compared to the amount of time, the category *words* and the unit *minute* are included as part of the rate.

1. George paid $3.99 for 12 doughnuts. Write the rate of cost in each of the following forms.

   a. using the word *per* **b.** using a colon
   c. using the fraction form **d.** using a slash (solidus)

2. A salesperson sold 7 cars in 10 days. Write the rate of sales in each of the following forms.

   a. using the word *per* **b.** using a colon
   c. using the fraction form **d.** using a slash (solidus)

3. Marcia knit 2 afghans in 5 weeks. Write the rate of production in each of the following forms.

   a. using the word *per* **b.** using a colon
   c. using the fraction form **d.** using a slash (solidus)

Check your answer by turning to the Appendix.

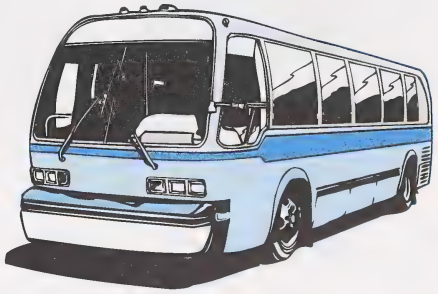
Comparing Quantities Expressed in Different Units

The world's most accurate mechanical clock is in the Copenhagen City Hall, Denmark. The clock is accurate to 0.5 seconds in 300 years. This means the clock will lose or gain no more than 0.5 seconds in 300 years.

A rate can be used to compare the number of seconds that could be lost or gained to the number of years.

Rates can be written in different ways.

Example



When you say that the bus from Edmonton to Calgary travelled a distance of 265 km in 3 h, you are comparing the amount of distance travelled to the amount of time required. Write the speed of the bus as a rate.

Solution

Method 1: Using the word *per*

The bus travelled at 265 km per 3 h.

Method 2: Using a colon

The bus travelled at 265 km : 3 h.

This is read as "265 km per 3 h."

Method 3: Using the fraction form

The bus travelled at $\frac{265 \text{ km}}{3 \text{ h}}$.

This is read as "265 km per 3 h."

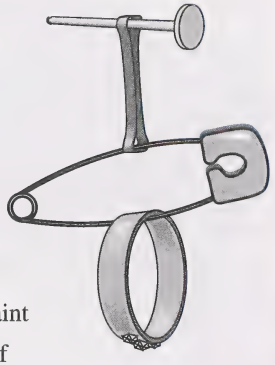
Method 4: Using a slash (solidus)

solidus

The bus travelled at 265 km/3 h.

This is read as "265 km per 3 h."

Note: Because the quantities being compared have different units, the units are included as part of the rate.



4. A rubber band stretched 1 cm when a 25-g mass was hung on it. Write the rate of stretch in each of the following forms.

- **a.** using the word *per*
- **b.** using a colon
- **c.** using the fraction form
- **d.** using a slash (solidus)

5. An advertisement stated that 4 L of paint would cover 25 m^2. Write the rate of coverage in each of the following forms.

- **a.** using the word *per*
- **b.** using a colon
- **c.** using the fraction form
- **d.** using a slash (solidus)

Check your answers by turning to the Appendix.

Simplifying Rates

In this module you have found that ratios are simplified by expressing them with the lowest whole-number terms possible.

Rates are usually simplified by expressing them with a second term of 1. Often the second term of 1 is merely understood and it is not written.

Example 1

A department store is having a shoe sale. To increase sales, it advertises the cost of $10.00 for 8 pairs of socks as a bonus. The cost of each pair of socks can be written as a rate in simplest form.

$$\frac{\text{cost (\$)}}{\text{number of pairs}}$$

$$\frac{10.00}{8} \overset{\div 8}{=} \frac{1.25}{1}$$

The cost is $1.25 per pair of socks.

Example 2

There was 81.6 mm of rain in 48 h. This relationship can be written as a rate in simplest form.

$$\frac{\text{amount of rain (mm)}}{\text{amount of time (h)}}$$

$$\frac{81.6}{48} \overset{\div 48}{=} \frac{1.7}{1}$$

The rate of rainfall was 1.7 mm/h.

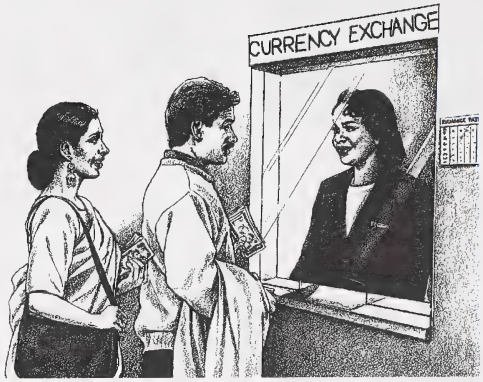
6. Write each of the following statements as rates in simplest form.

a. Bobby can type 420 words in 4 min.

b. Evan's heart beats 300 times in 4 min.

c. A rubber band stretches 2 cm when supporting a mass of 100 g.

d. Janis exchanged 225 Canadian dollars for 100 British pounds.



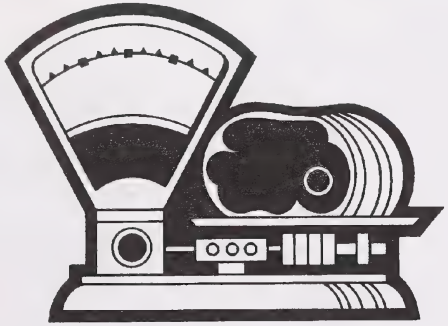
Check your answers by turning to the Appendix.

Proportional Rates

If you think of a rate as a rule, it is possible to write many proportional or equivalent rates.

Example

A supermarket sells roast beef for the price of $4.30 per kilogram.



An infinite number of rates can be written using this rule.

- The cost could be $8.60 per 2 kg.

$$\frac{\text{cost (\$)}}{\text{mass (kg)}} \qquad \frac{4.30}{1} \overset{\times 2}{=} \frac{8.60}{2}$$

- The cost could be $12.90 per 3 kg.

$$\frac{\text{cost (\$)}}{\text{mass (kg)}} \qquad \frac{4.30}{1} \overset{\times 3}{=} \frac{12.90}{3}$$

Note: The ratios in the example are proportional. The equations $\frac{4.30}{1} = \frac{8.60}{2}$ and $\frac{4.30}{1} = \frac{12.90}{3}$ are proportions.

7. Rita is a construction worker. She earns $8.20 per hour. Write three proportional rates to describe her rate of pay.

8. Arlene can swim 2.5 m per second. Write three proportional rates to describe her rate of speed swimming.

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

9. How can you arrange six toothpicks to form four congruent triangles? **Note:** You cannot break or bend the toothpicks.

Congruent triangles are triangles that have the same size and shape.

Check your answers by turning to the Appendix.

In this activity you wrote rates to compare the number of people, animals, or things in a group to a quantity. You wrote rates to compare quantities expressed in two different units. You continued solving non-routine problems.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you were shown one method of writing proportional ratios and rates. If you had difficulty simplifying ratios or rates using this method, you may find the following method helpful.

Example 1

Henri bought a package of 8 wieners and a package of 12 buns. What was the ratio of the number of buns to the number of wieners? Express the ratio in simplest form.

Solution

number of buns to number of wieners

$$12 : 8$$

$$\div 4 \qquad \div 4$$

$$= 3 : 2$$

The ratio of the number of buns to the number of wieners was 3 to 2.

Example 2

In order to make 900 mL of cooked pasta, you use 300 mL of uncooked pasta. What is the ratio of the amount of uncooked pasta to the amount of cooked pasta? Express the ratio in simplest form.

Solution

amount of uncooked pasta to amount of cooked pasta

$$300 : 900$$

$$\div 300 \quad \div 300$$

$$= 1 : 3$$

The ratio of the amount of uncooked pasta to the amount of cooked pasta is 1 to 3.

Example 3

Sally earned $56 in 8 h. What was her rate of pay? Express the rate in simplest form.

Solution

amount of earnings ($) per amount of time (h)

$$56 : 8$$

$$\div 8 \quad \div 8$$

$$= 7 : 1$$

Sally's rate of pay was $7 per hour.

Examples 1 to 3 show some of the differences between a ratio and a rate.

- A ratio is simplified by expressing the terms as the lowest possible whole numbers; a rate is simplified by expressing the second term as 1.
- Units are included in a rate; units are not included in a ratio.

1. Write a ratio in simplest form to describe each of the following situations.

a. At a County Fair, 20 student tickets were sold for every 15 adult tickets sold. What is the ratio of the number of student tickets sold to the number of adult tickets sold?

b. In a music store, CDs sell for $16 and cassettes sell for $10. What is the ratio of the price of CDs to the price of cassettes?

2. Write a rate in simplest form to describe each of the following situations.

a. It rained 42 mm in 10 h. What is the rate of rainfall?

b. Charlotte paid 28 Canadian dollars for 20 U.S. dollars. What is the exchange rate?

Check your answers by turning to the Appendix.

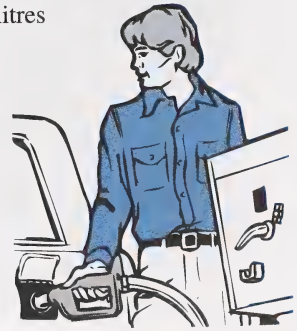
Enrichment

Not all rates are simplified by writing them with a second term of 1. Some rates are written with a second term of 100.

Example 1

Fuel consumption may be measured in litres per hundred kilometres.

If Gaston's car used 68 L in a 500-km trip, what was his rate of fuel consumption? Express this rate with a second term of 100.



Solution

$$\frac{\text{fuel consumed (L)}}{\text{distance travelled (km)}}$$

$$\frac{68}{500} \overset{\div 5}{=} \frac{13.6}{100}$$

Gaston's gas consumption is 13.6 L/100 km.

Example 2

The unit cost of some products is measured in dollars per hundred millilitres or dollars per hundred grams.

If shampoo costs $4.20 for 500 mL, what is the unit rate of cost? Express this rate with a second term of 100.

Solution

$$\frac{\text{cost (\$)}}{\text{volume (mL)}}$$

$$\frac{4.20}{500} \overset{\div 5}{=} \frac{0.84}{100}$$

The unit cost is $0.84/100 mL.

1. If Manuella's car used 72 L of gas in a 800-km trip, what was the fuel consumption rate? Express this rate with a second term of 100 km.

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2. If Ruby paid $6.50 for 500 g of smoked turkey, what was the unit rate of cost? Express this rate with a second term of 100 g.

3. If Denzil paid $1.29 for a 1-L bottle of pop, what was the unit rate of cost? Express this rate with a second term of 100 mL.
Hint: 1 L = 1000 mL.

Check your answers by turning to the Appendix.

In this section you have distinguished between a ratio and a rate. Usually the comparison of the number of people, animals, or things in different groups is expressed as a ratio. However, when the comparison uses the word *per*, it is sometimes referred to as a rate.

The birth rate, marriage rate, and divorce rate are expressed as the number of births, marriages, or divorces per 1000 population.

Example 3

There were 185 597 marriages in Canada in 1985. If the population in 1985 was about 25 360 000, what was the marriage rate?

Solution

$$\frac{\text{number of marriages}}{\text{total number of people}}$$

$$\frac{185\,597}{25\,360\,000} \overset{\div 25\,360}{\doteq} \frac{7.32}{1000}$$

In 1985 the marriage rate in Canada was about 7.32 marriages per 1000 population.

4. There were 61 980 divorces in Canada in 1985. If the population in 1985 was about 25 360 000, what was the divorce rate?

Check your answers by turning to the Appendix.

Now Try This

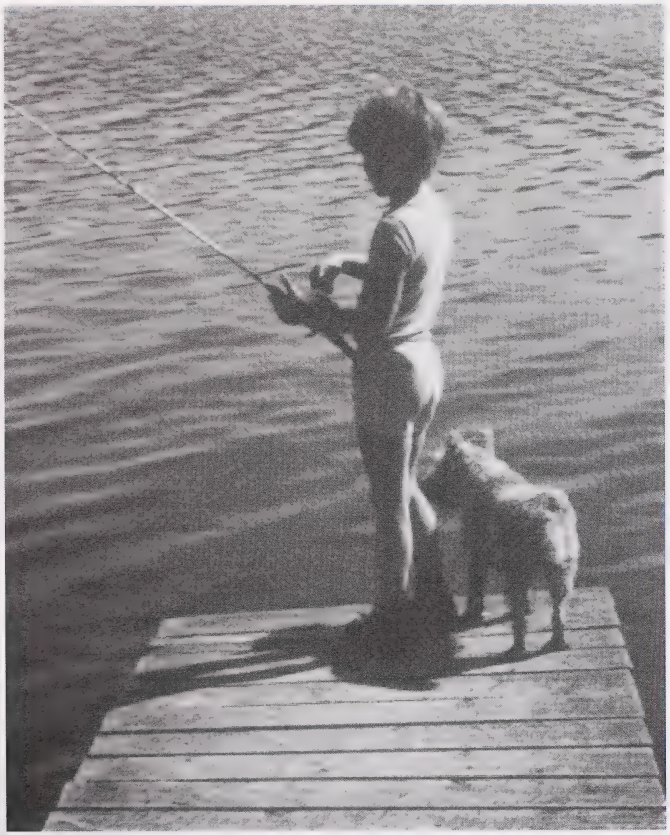
Statistics Canada has a home page on the Internet. This is its uniform resource locator (URL).

http://www.statcan.ca/start.html

To find the latest statistics on the Canadian birth rates, marriage rates, and divorce rates, go the home page, click on "Frequently Asked Statistics," and then click on "Various Social Indicators."

Are the birth rates, marriage rates, and divorce rates increasing or decreasing?

Conclusion



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In this section you distinguished between a ratio and a rate. You wrote ratios in different ways: using the word *to*, using a colon, and using the fraction form. You wrote rates in different ways: using the word *per*, using a colon, using the fraction form, and using a slash (solidus). You simplified ratios and rates, and you wrote proportional ratios or rates.

You investigated many different kinds of ratios and rates. You discovered that the number of heartbeats in one minute is known as the resting rate. The heart rate changes with the activity level. How do you think your heart rate while fishing compares to your heart rate after vigorous exercise?

Assignment

Assignment Booklet

You are ready to do the module assignment for Section 1.

Section 2: Percents and Other Ratio Forms

Have you ever watched a bee in a flower garden? Did you know that a bee is very strong? It is estimated that a bee can pull an object having a mass 300 times the bee's own body mass. This means a bee could pull a mass that is 30 000% of its own body mass.

In this section you will discover some other interesting ratios.

You will express a ratio with the first term less than the second term as a percent, as a fraction, and as a decimal number. You will write a ratio with equal terms as 100% or 1. You will convert a ratio with the first term greater than the second term to a percent, to a whole number or mixed number, and to a decimal number.

Activity 1: Percents Less than 100%

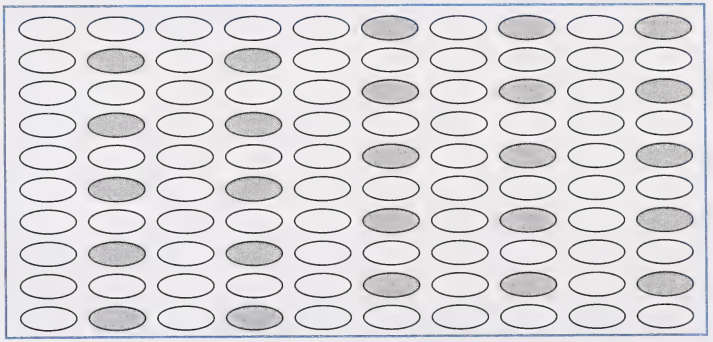
Did you know that the Inuit have many words to describe snow?

Just as every language has synonyms to describe something, mathematics has many equivalent ways to describe a relationship.

In this activity you will investigate ratios where the first term is less than the second term. You will write each ratio as a fraction, as a decimal number, and as a **percent**.

A percent is a special ratio with 100 as the second term. Percent means per hundred.

Example 1



In the given diagram, there is a group of 25 shaded ovals.

The ratio of the number of shaded ovals to the total number of ovals is 25 to 100. Express this ratio as a fraction, as a decimal number, and as a percent.

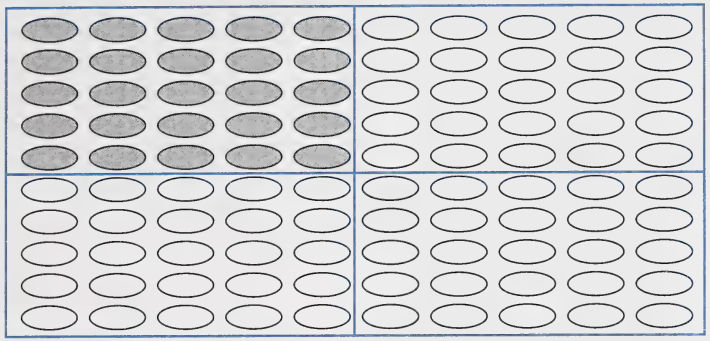
Solution

As a fraction

$$\frac{\text{number of shaded ovals}}{\text{total number of ovals}} \qquad \frac{25}{100} = \frac{1}{4}$$

In the diagram, $\frac{1}{4}$ of the total number of ovals are shaded.

Rearranging the ovals may help you visualize the fraction form.



In the diagram, $\frac{1}{4}$ of the total number of ovals are shaded.

As a decimal number

$$\frac{\text{number of shaded ovals}}{\text{total number of ovals}} \qquad \frac{25}{100} = 0.25$$

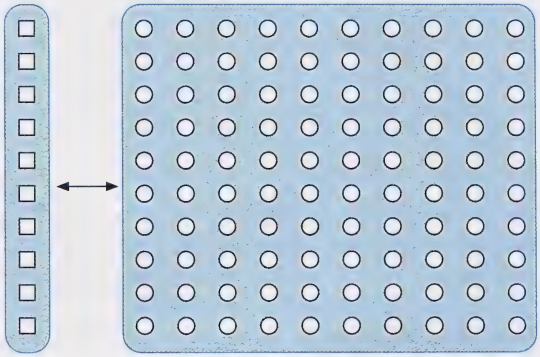
In the diagram, 0.25 of the total number of ovals are shaded.

As a percent

$$\frac{\text{number of shaded ovals}}{\text{total number of ovals}} \qquad \frac{25}{100} = 25\%$$

In the diagram, 25% of the total number of ovals are shaded.

Example 2



There are two groups of shapes in the given diagram: a group of 10 squares and a group of 100 circles.

The ratio of the number of squares to the number of circles is 10 to 100. Express this ratio as a fraction, as a decimal number, and as a percent.

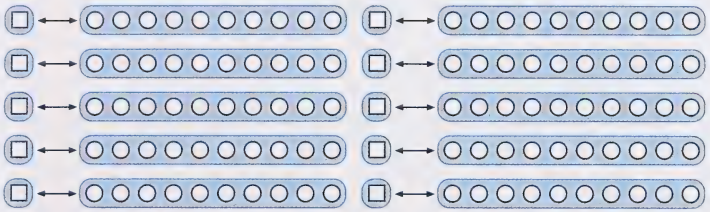
Solution

As a fraction

$$\frac{\text{number of squares}}{\text{number of circles}} \qquad \frac{10}{100} = \frac{1}{10}$$

The number of squares in the diagram is $\frac{1}{10}$ of the number of circles.

Regrouping the squares and circles may help you visualize the fraction form.



There is 1 square for every 10 circles in the diagram.

As a decimal number

$$\frac{\text{number of squares}}{\text{number of circles}} \qquad \frac{10}{100} = 0.10$$
$$= 0.1$$

The number of squares in the diagram is 0.1 of the number of circles.

As a percent

$$\frac{\text{number of squares}}{\text{number of circles}} \qquad \frac{10}{100} = 10\%$$

The number of squares in the diagram is 10% of the number of circles.

1. A bowl of raisin bran cereal contains 20 raisins to 100 bran flakes. Express this relationship in the following ways:

a. as a fraction in simplest form
b. as a decimal number
c. as a percent

2. The ratio of the length of a grasshopper's body to the distance the grasshopper can jump is 5 to 100. Express this relationship in the following ways:

a. as a fraction in simplest form
b. as a decimal number
c. as a percent

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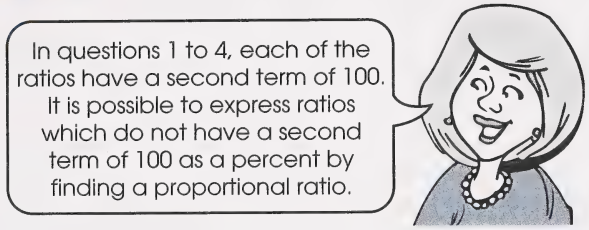
3. The ratio of the mass of an ant's own body to the mass of the object the ant can lift is 2 to 100. Express this relationship in the following ways:

a. as a fraction in simplest form
b. as a decimal number
c. as a percent

4. In yesterday's basketball game, the Griffins made 35 baskets out of 100 attempts. Express this relationship in the following ways:

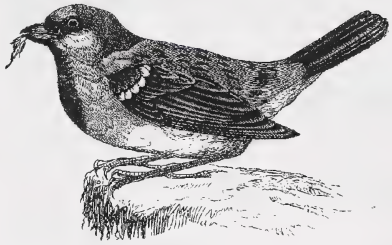
   a. as a fraction in simplest form
   b. as a decimal number
   c. as a percent

Check your answers by turning to the Appendix.



Example 3

The ratio of the time for a sparrow's egg to hatch to the time for a penguin's egg to hatch is 1 to 5.



Express this ratio as a fraction, as a decimal number, and as a percent.

Solution

As a fraction

$$\frac{\text{time for hatching a sparrow egg}}{\text{time for hatching a penguin egg}} \qquad \frac{1}{5}$$

The time for a sparrow egg to hatch is $\frac{1}{5}$ of the time for a penguin egg to hatch.

As a decimal number

$$\frac{\text{time for hatching a sparrow egg}}{\text{time for hatching a penguin egg}} = \frac{1}{5} = \frac{2}{10} = 0.2$$

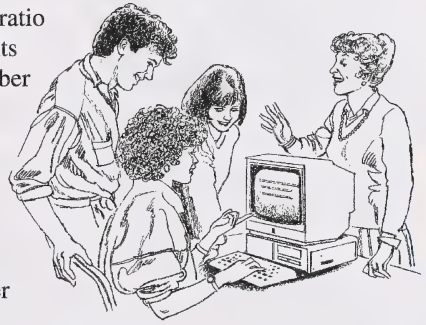
The time for a sparrow egg to hatch is 0.2 of the time for a penguin egg to hatch.

As a percent

$$\frac{\text{time for hatching a sparrow egg}}{\text{time for hatching a penguin egg}} = \frac{1}{5} = \frac{20}{100} = 20\%$$

The time for a sparrow egg to hatch is 20% of the time for a penguin egg to hatch.

5. In a computer class the ratio of the number of students present to the total number of students is 3 to 4. Express this ratio in the following ways:



a. as a fraction in simplest form
b. as a decimal number
c. as a percent

6. The ratio of the number of pages Bruce read to the number of pages Tony read is 4 to 5. Express this ratio in the following ways:

a. as a fraction in simplest form
b. as a decimal number
c. as a percent

7. The ratio of the length of a nail to the length of a pencil is 3 to 5. Express the ratio in the following ways:

a. as a fraction in simplest form
b. as a decimal number
c. as a percent

Check your answers by turning to the Appendix.

Expressing Decimal Numbers as Percents

Sometimes you may wish to change a ratio expressed as a decimal number to its percent form.

Example

The 1982 Canadian gold coin is an alloy of gold and silver; 0.92 of the coin is gold. Express this decimal number as a percent.

Solution

Step 1: Write the decimal number as an equivalent ratio with a second term of 100.

$$0.92 = \frac{92}{100}$$

Step 2: Write the equivalent ratio as a percent.

$$\frac{92}{100} = 92\%$$

So, 92% of the 1982 gold coin is gold.

8. Advance tickets to a concert are sold at 0.75 of the cost at the door. Write this decimal number as a percent.

9. During the 1914 baseball season, Ty Cobb had a batting average of 0.390. This means he hit safely 0.390 of the times he was up to bat. Write the decimal number as a percent.

Check your answers by turning to the Appendix.

Expressing Fractions as Percents

Sometimes you may wish to change a ratio expressed as a fraction to its percent form.

Example

Mrs. Ronet and Mrs. Martinez are neighbours. Mrs. Ronet's monthly household expenses are $\frac{4}{5}$ of Mrs. Martinez's expenses. Express this fraction as a percent.

Solution

Step 1: Find an equivalent ratio with 100 as the second term.

$$\frac{4}{5} \overset{\times 20}{=} \frac{80}{100}$$

Step 2: Write the equivalent ratio as a percent.

$$\frac{80}{100} = 80\%$$

Mrs. Ronet's expenses are 80% of Mrs. Martinez's.

10. About $\frac{3}{4}$ of the pets in Wasylena's neighbourhood are dogs. Express this fraction as a percent.

11. Mr. Sabban sowed $\frac{1}{5}$ of his fields in barley. Express this fraction as a percent.

Check your answers by turning to the Appendix.

Expressing Percents as Decimal Numbers and Fractions

It is very useful to be able to express percents less than 100% as decimal numbers and fractions.

Example

An alloy of copper, zinc, and lead is used to make the parts of a watch. Copper is 64% of the alloy. Express this ratio as a decimal number and a fraction.

Solution

As a decimal number

$$64\% = \frac{64}{100}$$ ← Write the percent as a ratio with a second term of 100.

$$= 0.64$$ ← Write the ratio as a decimal number.

Copper is 0.64 of the alloy.

As a fraction

$$64\% = \frac{64}{100}$$ ← Write the percent as a ratio with a second term of 100.

$$= \frac{16}{25}$$ ← Write the ratio as a fraction in simplest form.

Copper is $\frac{16}{25}$ of the alloy.

12. The elementary school students sold chocolate bars to raise money. They kept 20% of the sales. Express this percent in each of the following forms.

a. as a decimal number
b. as a fraction in simplest form

13. A metal alloy is made of zinc, tin, and lead. The amount of tin is 60% of the amount of lead. Express this percent in each of the following forms.

a. as a decimal number **b.** as a fraction in simplest form

14. In a basketball game, Darryl made 35% of the shots he attempted. Express this percent in each of the following forms.

a. as a decimal number
b. as a fraction in simplest form

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

15. Erika held out a closed hand. "I have $1.13 and there are 12 coins in my hand," she said.

What are the coins in Erika's hand?

Check your answer by turning to the Appendix.

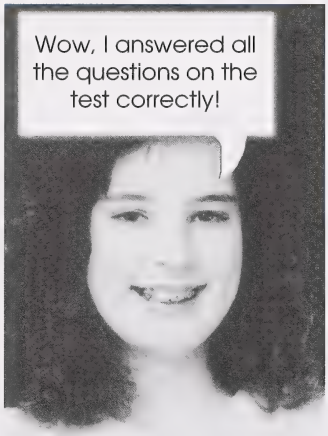
In this activity, you examined ratios with a first term less than a second term. You wrote these ratios as fractions, as decimal numbers, and as percents. You expressed decimal numbers and fractions as percents. You changed percents to fractions and decimal numbers.

All the percents you examined in this activity were less than 100%. In the next activity you will discover the meaning of 100%.

Activity 2: Percents Equal to 100%

There were 25 questions on a test and Ramona answered all 25 questions correctly.

The ratio of the number of questions Ramona answered correctly to the total number of questions is 25 to 25. In other words, the ratio of the number of questions Ramona answered correctly to the total number of questions is 1 to 1.



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This ratio can also be expressed as a whole number.

$$\frac{\text{number of questions answered correctly}}{\text{total number of questions}} = \frac{25}{25} = \frac{1}{1} = 1$$

So, the number of questions answered correctly was 1 times the total number of questions.

This ratio can be expressed as a percent.

$$\frac{\text{number of questions answered correctly}}{\text{total number of questions}} = \frac{25}{25} = \frac{1}{1} = \frac{100}{100} = 100\%$$

So, Ramona answered 100% of the questions correctly.

Examples 1 and 2 give situations where the terms of the ratio are equal.

Example 1

Container A has 750 mL of popcorn. Container B has 750 mL of popcorn. So, the ratio of the amount of popcorn in Container A to the amount of popcorn in Container B is 750 to 750, or 1 to 1. Express the ratio as a whole number and as a percent.

Solution

As a whole number

$$\frac{\text{amount of popcorn in A (mL)}}{\text{amount of popcorn in B (mL)}} = \frac{750}{750} = \frac{1}{1} = 1$$

The amount of popcorn in Container A is 1 times the amount of popcorn in Container B.

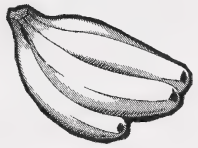
As a percent

$$\frac{\text{amount of popcorn in A (mL)}}{\text{amount of popcorn in B (mL)}} = \frac{750}{750} = \frac{1}{1} = \frac{100}{100} = 100\%$$

The amount of popcorn in Container A is 100% of the amount of popcorn in Container B.

Example 2

There are 5 bananas in a bowl. All 5 bananas are ripe. So, the ratio of the number of ripe bananas to total number of bananas is 5 to 5, or 1 to 1. Express this ratio as a whole number and as a percent.



Solution

As a whole number

$$\frac{\text{number of ripe bananas}}{\text{total number of bananas}} = \frac{5}{5} = \frac{1}{1} = 1$$

So, the number of ripe bananas is 1 times the number of bananas in the bowl.

As a percent

$$\frac{\text{number of ripe bananas}}{\text{total number of bananas}} = \frac{5}{5} = \frac{1}{1} = \frac{100}{100} = 100\%$$

So, 100% of the bananas in the bowl are ripe.

1. A cranapple drink is made with 375 mL of cranberry juice and 375 mL of apple juice. What is the ratio of the amount of cranberry juice to the amount of apple juice?

a. Express the ratio in lowest terms.
b. Express the ratio as a whole number.
c. Express the ratio as a percent.

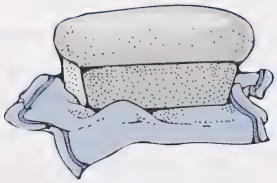
2. Henri opened a carton of 12 eggs and found 12 eggs were cracked. What is the ratio of the number of cracked eggs to the total number of eggs?

a. Express the ratio in lowest terms.
b. Express the ratio as a whole number.
c. Express the ratio as a percent.

Check your answers by turning to the Appendix.

You can think of a whole as 100%.

Example 3



You can purchase bread that is 60% whole wheat. What percent of the flour is white?

Solution

The flour used to make 60% whole-wheat bread is a blend of white flour and whole wheat flour; 60 parts out of 100 parts of flour are whole-wheat flour.

If all the flour was whole-wheat flour, it would be 100% whole wheat.

$$100\% - 60\% = 40\%$$

So, 40% of the flour is white.

Note: This means 40 out of 100 parts of flour is white.

3. Ramon won 30% of the ski races he entered. What percent of the races did he lose?

4. In a group of children, 5% are left-handed. What percent are right-handed?

5. In a certain group of men, 20% wear glasses. What percent do not wear glasses?

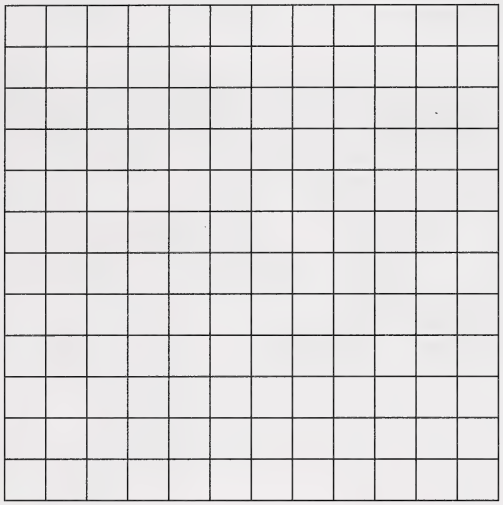
6. A box of stars contains stars that are red, blue, green, and yellow. If 30% of the stars are red, 25% are blue, and 15% are green, what percent are yellow?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

7. What is the greatest number of small squares that can be shaded in this diagram so that no two squares are in the same row, column, or diagonal?

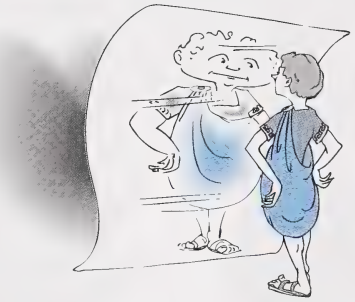


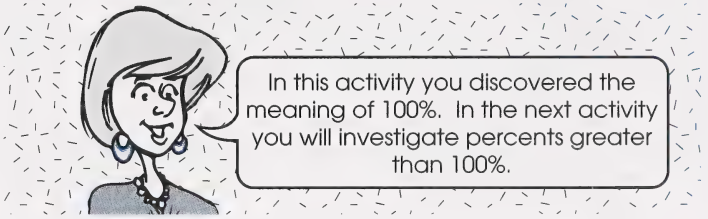
Check your answers by turning to the Appendix.

Did You Know?

Mathematicians Making Things[1]

Mathematicians have made some very unusual things. Hero of Alexandria (Heron), who lived around A.D. 70, fixed the temple doors so that they could open as supermarket doors open today. He made a mirror in which a person could see the back of his or her head. He even made mirrors that showed a person upside down, very fat, or very thin. People loved to see themselves in his mirrors.





[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*, by Merle Mitchell. Copyright 1978 by the National Council of Teachers of Mathematics.

Activity 3: Percents Greater than 100%

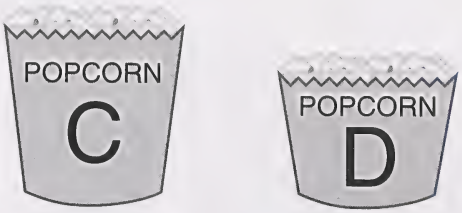
Did you know that the cheetah is the fastest land animal in the world over short distances? The ratio of the speed of a cheetah to the speed of a jackal is 2 to 1. This means that the cheetah is twice as fast as the jackal. You can also say that the speed of the cheetah is 200% the speed of the jackal.

In this activity you will investigate other ratios where the first term is greater than the second term. You will express each ratio as a whole number or a mixed number and as a percent.

Expressing Ratios as Percents and Whole Numbers

Example

Container C has 750 mL of popcorn. Container D has 375 mL of popcorn. So, the ratio of the amount of popcorn in Container C to the amount of popcorn in Container D is 750 to 375, or 2 to 1. Express the ratio as a whole number and as a percent.



Solution

As a whole number

$$\frac{\text{amount of popcorn in C (mL)}}{\text{amount of popcorn in D (mL)}} \qquad \frac{750}{375} = \frac{2}{1}$$
$$= 2$$

The amount of popcorn in Container C is 2 times the amount of popcorn in Container D.

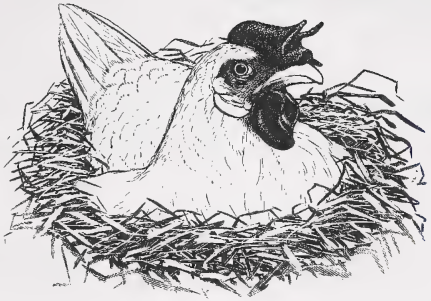
As a percent

$$\frac{\text{amount of popcorn in C (mL)}}{\text{amount of popcorn in D (mL)}} \quad \frac{750}{375} = \frac{2}{1} = \frac{200}{100} = 200\%$$

The amount of popcorn in Container C is 200% of the amount of popcorn in Container D.

1. This year a hen laid 156 eggs. Last year the hen laid 52 eggs. What is the ratio of the number of eggs laid this year to the number of eggs laid last year? Express this ratio in the following ways.

a. as a ratio with the word *to*
b. as a whole number
c. as a percent



2. On Earth a high jumper can jump 2 m. Scientists estimate that on the Moon the same high jumper could jump 12 m. What is the ratio of the height of the jump on the Moon to the height of the jump on Earth? Express this ratio in the following ways.

a. as a ratio using a colon
b. as a whole number
c. as a percent

Check your answers by turning to the Appendix.

Expressing Ratios as Mixed Numbers, Decimal Numbers, and Percents

Now that you examined some ratios that can be expressed as whole numbers, you will investigate some ratios that can be written as mixed numbers.

Example

Joan borrowed \$20 from her mother and repaid her \$22. So, the ratio of the amount of money Joan repaid her mother to the amount of money she borrowed is 22 to 20. Express this ratio as a mixed number, as a decimal number, and as a percent.

Solution

As a mixed number

$$\frac{\text{amount repaid (\$)}}{\text{amount borrowed (\$)}} \qquad \frac{22}{20} = \frac{11}{10}$$
$$= 1\frac{1}{10}$$

Joan repaid $1\frac{1}{10}$ of the money she borrowed.

As a decimal number

$$\frac{\text{amount repaid (\$)}}{\text{amount borrowed (\$)}} \qquad \frac{22}{20} = \frac{11}{10}$$
$$= 11 \div 10$$
$$= 1.1$$

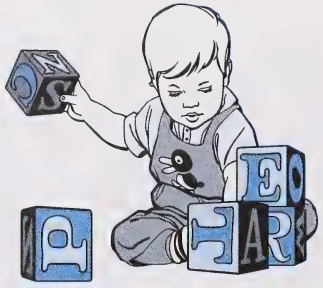
Joan repaid 1.1 times the money she borrowed.

As a percent

$$\frac{\text{amount repaid (\$)}}{\text{amount borrowed (\$)}} \qquad \frac{22}{20} = \frac{11}{10}$$
$$= \frac{110}{100}$$
$$= 110\%$$

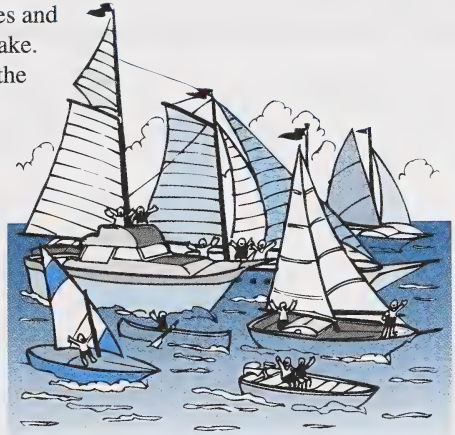
Joan repaid 110% of the money she borrowed.

3. Bobby's height was 60 cm last year. Today his height is 75 cm. What is the ratio of his present height to his height last year? Express this ratio in the following ways.



a. as a ratio with the word *to*
b. as a mixed number
c. as a decimal number
d. as a percent

4. There were 24 canoes and 15 sailboats on the lake. What is the ratio of the number of canoes to the number of sailboats? Express this ratio in each of the following ways.



a. as a ratio with the word *to*
b. as a mixed number
c. as a decimal number
d. as a percent

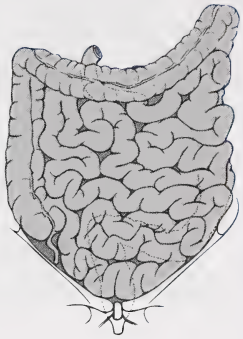
Check your answers by turning to the Appendix.

Expressing Whole Numbers, Mixed Numbers, and Decimal Numbers as Percents

Sometimes you may wish to change a ratio in whole number form or mixed number form to a percent.

Example 1

The length of a person's small intestine is 4 times the length of a person's large intestine. Express this whole number as a percent.



Solution

$$4 = \frac{4}{1}$$
$$= \frac{400}{100}$$
$$= 400\%$$

The length of the small intestine is 400% of the length of the large intestine.

Example 2

The speed at which the canvasback duck can fly is 2.3 times the speed at which an ostrich can run. Express this ratio as a percent.

Solution

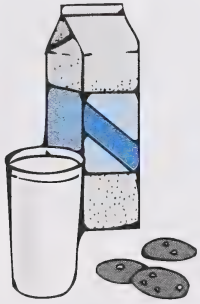
$$2.3 = \frac{2.3}{1}$$
$$= \frac{230}{100}$$
$$= 230\%$$

The speed at which a canvasback duck can fly is 230% of the speed at which an ostrich can run.

5. Some jet planes travel at a speed that is about 6 times the flying speed of the spine-tail swift. Express this ratio as a percent.

6. A supersonic jet can fly at a speed that is 2 times the speed of sound. Express this ratio as a percent.

7. The mass of milk is 1.56 times the mass of gasoline. Express this ratio as a percent.



8. The mass of an ostrich egg is 1 000 000 times the mass of a hummingbird egg. Express this ratio as a percent.

9. The diameter of the wire to make coat hangers is 7.5 times the diameter of a human hair. Express this ratio as a percent.

Check your answers by turning to the Appendix.

Expressing Percents as Whole Numbers, Mixed Numbers, and Decimal Numbers

It is very useful to be able to express percents greater than 100% as whole numbers, mixed numbers, and decimal numbers.

Example 1

Lake Superior is the largest of the Great Lakes. Lake Ontario is the smallest. The area of Lake Superior is 400% of the area of Lake Ontario. Express this percent as a whole number.

Solution

As a whole number

$$400\% = \frac{400}{100}$$
$$= 4$$

Lake Superior is 4 times the size of Lake Ontraio.

Example 2

The Smiths sold their house for 125% of the purchase price. Express this percent as a mixed number and a decimal number.

Solution

As a mixed number

$$125\% = \frac{125}{100}$$
$$= 1\frac{25}{100}$$
$$= 1\frac{1}{4}$$

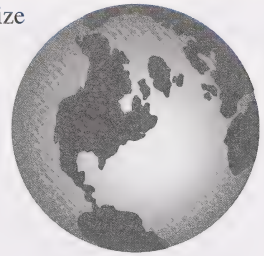
The Smiths sold their house for $1\frac{1}{4}$ times the purchase price.

As a decimal number

$$125\% = \frac{125}{100}$$
$$= 1.25$$

The Smiths sold their house for 1.25 times the purchase price.

10. **a.** The size of Earth is 700% of the size of Mars. Express this percent as a whole number.



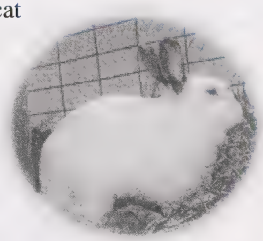
b. The size of Earth is 1600% of the size of Mercury. Express this percent as a whole number.

11. The volume of steam is 167 000% of the volume of water. Express this percent as a whole number.

12. The average life span of a domestic sheep is 150% of the life span of a domestic goat. Express this percent in each of the following forms.

a. as a mixed number (in simplest form)
b. as a decimal number

13. The average life span of a domestic cat is 240% of the life span of a domestic rabbit. Express this percent in each of the following forms.



a. as a mixed number (in simplest form)

b. as a decimal number

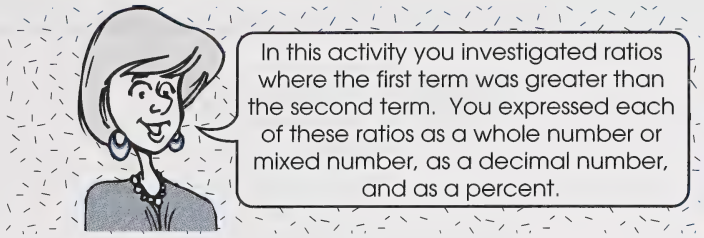
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

14. One wall in a museum is 10 m long. Picture hooks are to be placed every metre along the wall. How many hooks will be needed?

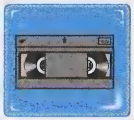
Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help



If you had difficulty understanding percents, view the video program entitled *Percents* from the series *Mathways*.

Note: In the video, the word *percent* is spelled as two separate words; this is an alternative spelling. You may ignore the discussion of fractional percents such as $33\frac{1}{3}\%$ or $12\frac{1}{2}\%$. You will deal with these percent forms in a later course.

1. What does 1% mean?

2. What does 250% mean?

Check your answers by turning to the Appendix.

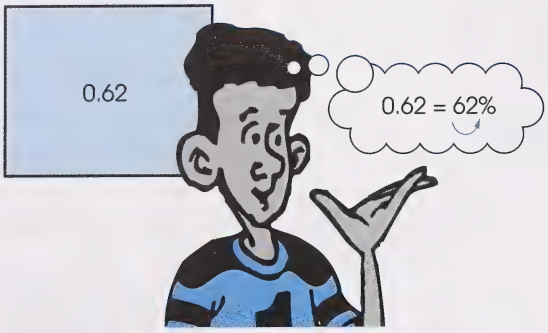
Enrichment

Do you know a shortcut method to change a decimal number to a percent? If not, study the following examples and look for a pattern.

Example 1

$\therefore\ 0.75 = 75\%$

Example 2



$\therefore\ 0.62 = 62\%$

Example 3

$\therefore\ 0.05 = 5\%$

Example 4

$\therefore\ 3 = 300\%$

Example 5

$\therefore\ 3.5 = 350\%$

1. Use mental math to complete the following table.

Number	Percent
0.60	
0.35	
0.08	
2	
6.2	

Check your answers by turning to the Appendix.

Do you know a shortcut method to change a percent to a decimal number? If not, study the following examples and look for a pattern.

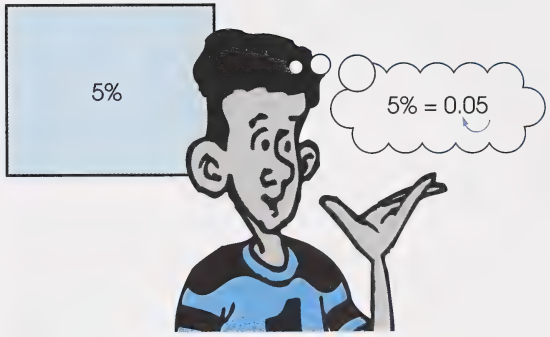
Example 6

$\therefore\ 80\% = 0.8$

Example 7

$\therefore\ 125\% = 1.25$

Example 8



$\therefore\ 5\% = 0.05$

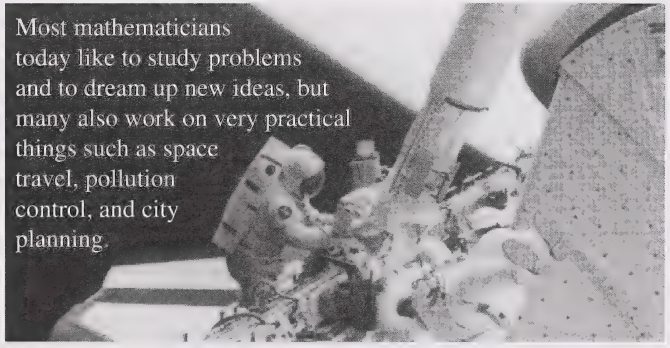
2. Use mental math to complete the given table.

Percent	Number
50	
8	
62	
400	
375	

Check your answers by turning to the Appendix.

Did You Know?

Mathematicians Making Things[1]



NASA

[1] Reprinted with permission from *Mathematical History: Activities, Puzzles, Stories, and Games*, by Merle Mitchell. Copyright 1978 by the National Council of Teachers of Mathematics.

Mathematicians in the past have also made some most unusual things.

Alhazen, an Arab mathematician who lived around A.D. 1000, was very proud of his knowledge of geometry and once boasted that he could make a machine to control the floods from the Nile River. The ruler in Egypt was interested in this machine and sent for Alhazen. The Arabian knew his idea wouldn't work, but he was afraid to tell the ruler. He pretended to have lost his mind so that he wouldn't be expected to make the machine. He learned his lesson about boasting because he had to go on pretending until the ruler died.

In the late 1400s, a mathematician named Johann Müller lived near a place in Germany called King's Mountain. Müller was nicknamed Regiomontanus, which is the Latin name of the town. Müller worked on the calendar, studied geometry, and wrote about astronomy. Most of the people of his day did not know about his mathematics, but they did enjoy seeing his mechanical eagle flying around the mountain with its big wings flapping.

Pascal was a French mathematician of the seventeenth century. He invented an adding machine. His machine could add six-digit numbers. He also invented the one-wheel wheelbarrow.

Conclusion

In this section you explored percents and other ratio forms. You discovered that when a number or an amount is compared to a greater number or a larger amount, the ratio can be expressed as a percent less than 100% or a number less than 1 (a fraction or decimal number). When a number or an amount is compared to an equal number or amount, the ratio can be expressed as 100% or 1. When a number or amount is compared to a lesser number or smaller amount, the ratio can be expressed as a percent greater than 100% or a number greater than 1 (a whole number, a mixed number, or a decimal number).

You discovered that a bee might pull a mass 30 000% of its own body mass! A bee weighs about 0.1 g. This means that a bee would pull a mass of about 30 g. How much mass might you pull if you were a bee? If you weigh 50 kg, you would be able to pull a mass of 15 tonnes!

You may enjoy investigating other ratios and percents to find more interesting facts.

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 2.

Section 3: Proportions

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Do you enjoy watching a train? Have you taken a trip on the train? Have you ever had a model train set?

In western Canada trains are often used to haul freight. The heaviest trains haul coal, sulphur, potash, and grain. The current maximum load is 118 tonnes per car. The CN and CP railways are considering a new load limit of 120.6 tonnes per car. At this rate, what would be the load limit of a 50-car train?

To solve this problem you need to use proportional reasoning.

In this section you will compare two ratios or two rates. You will also use proportions to solve problems involving ratios, rates, and percents.

Activity 1: Comparing Ratios and Rates

At one store bananas cost $7.35 for 5 kg. In another store bananas cost $4.41 for 3 kg. Which store is offering the better deal?

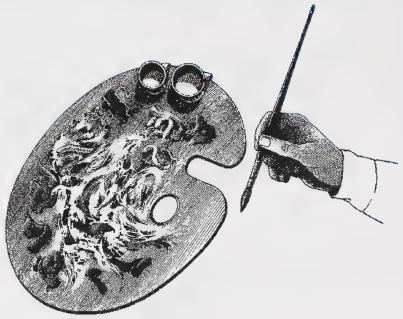
To compare the prices, write the rates in simplest form.

	Store 1	Store 2
$\frac{\text{price (\$)}}{\text{mass (kg)}}$	$\frac{7.35}{5} = \frac{1.47}{1}$	$\frac{4.41}{3} = \frac{1.47}{1}$

Since the rates in simplest form are each equal to $1.47/kg, the stores are offering the same deal.

Comparing ratios and rates is a useful skill.

Example 1



Julia likes to paint. Will the ratio of 15 mL of blue to 10 mL of yellow paint produce the same colour of green as the ratio of 21 mL of blue paint to 14 mL of yellow paint?

Solution

Step 1: Write the ratios in simplest form.

	Green 1	Green 2
$\frac{\text{amount of blue paint (mL)}}{\text{amount of yellow paint (mL)}}$	$\frac{15}{10} = \frac{3}{2}$	$\frac{21}{14} = \frac{3}{2}$

Step 2: Compare the ratios.

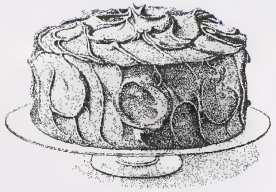
Since the ratios in simplest form are each equal to $\frac{3}{2}$, they are proportional.

So, the same green colour will be produced.

Note: Be careful of the order of terms when comparing ratios or rates.

Example 2

In a cake recipe the ratio of the number of parts of flour to the number of parts of sugar is 3:1. In another cake recipe the ratio of the number of parts of sugar to the number of parts of flour is 3:1. Which recipe makes sweeter cakes?



Solution

The recipe with the greater ratio of sugar to flour will be sweeter.

Step 1: The ratios in the problem are not written in the same order. Rewrite the ratios so that they are comparing the amount of sugar to the amount of flour.

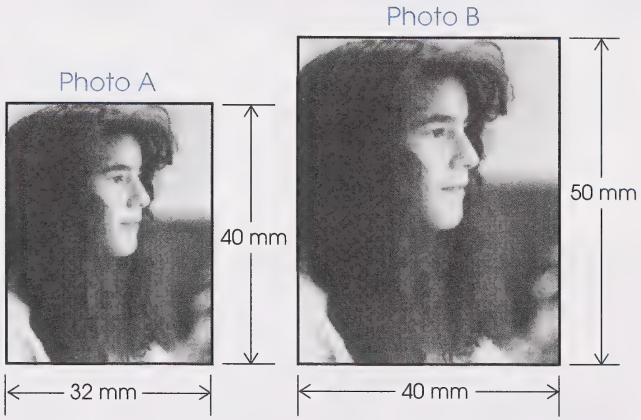
$\frac{\text{number of parts of sugar}}{\text{number of parts of flour}}$	Recipe 1	Recipe 2
	$\frac{1}{3}$	$\frac{3}{1}$

Step 2: Compare the ratios.

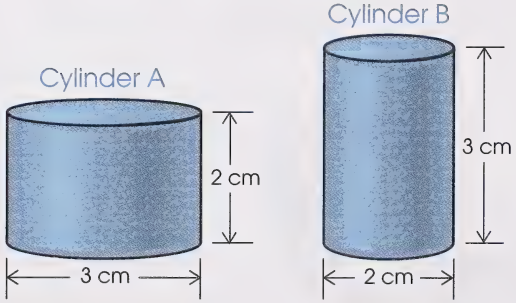
$$\frac{1}{3} < \frac{3}{1}$$

So, the second recipe is sweeter.

1. Look at the photographs. Compare the ratios of the lengths to the widths. Are they equivalent?



2. Look at these cylinders. Compare the ratios of the heights to the diameters. Are they equivalent?



3. Dino travelled 830 km in 9 h. Frank travelled 640 km in 7 h. Who travelled at a faster rate of speed?

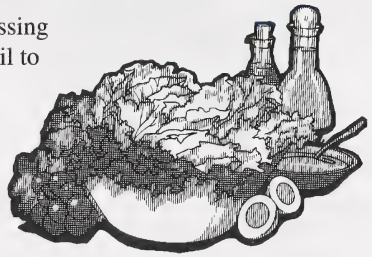
4. One athlete runs 100 m in 9.9 seconds. Another athlete runs 200 m in 19.8 seconds. Which athlete runs faster?

5. The following chart gives the quantities of rice required to serve 2, 4, or 6 people.

	Servings		
	2	4	6
Rice (mL)	125	175	250
Water (mL)	300	375	500
Butter (mL)	5	10	15
Salt (mL)	2	5	7

Is the ratio of the amount of rice to the amount of water proportional for 2, 4, and 6 servings? Answer **yes** or **no** and explain why.

6. In a certain kind of salad dressing there is a ratio of 2 parts of oil to 1 part of vinegar. Could the amounts in each of the following situations be the amount of oil and vinegar in the salad dressing? Answer **yes** or **no** and explain why.



a. 60 mL of oil and 30 mL of vinegar
b. 30 mL of oil and 60 mL of vinegar
c. 90 mL of oil and 45 mL of vinegar
d. 45 mL of oil and 90 mL of vinegar

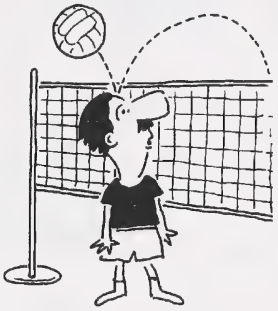
7. The following chart gives the cost for 2 kg, 4 kg, and 6 kg of ground beef. Are the rates proportional? Answer **yes** or **no** and explain why.

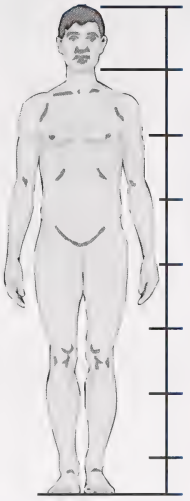
Cost ($)	8.60	17.20	25.80
Mass (kg)	2	4	6

Check your answers by turning to the Appendix.

Did You Know?

What do you notice about this cartoon character?





Did you notice that the cartoon character is not in proportion? The size of his head is too large.

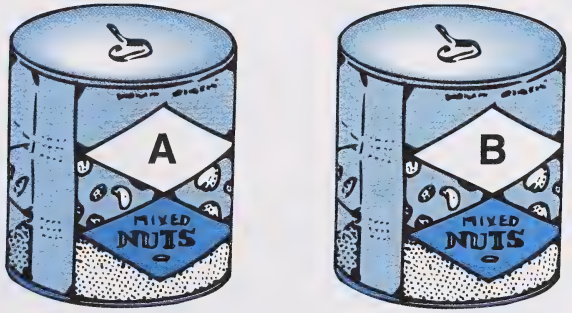
The ratio of the length of a man's head to his height is actually 1 to 7.5.

Using Percents to Compare

Another way to compare ratios is to write them in their percent forms.

Example

In Can A the ratio of the number of cashews to the total number of mixed nuts is 1 to 5. In Can B the ratio of the number of cashews to the total number of mixed nuts is 3 to 10. Which can has the higher ratio of cashews to mixed nuts?



Solution

Step 1: Write the ratios as percents.

$$\frac{\text{number of cashews}}{\text{total number of mixed nuts}}$$

Can A	Can B
$\frac{1}{5} = \frac{20}{100}$	$\frac{3}{10} = \frac{30}{100}$
$= 20\%$	$= 30\%$

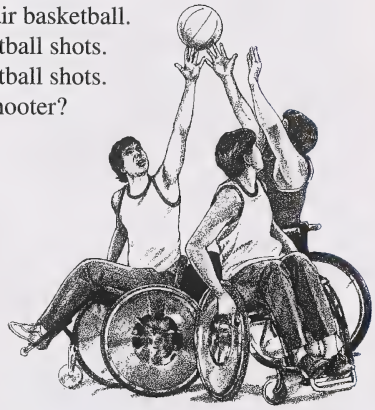
Step 2: Compare the percents.

$30\% > 20\%$

So, Can B has the higher ratio of cashews to mixed nuts.

8. Kara scored 17 out of 25 on the first test and 14 out of 20 on the second test. On which test did she do better?

9. Matt and Jon play wheelchair basketball.
Matt sank 7 out of 10 basketball shots.
Jon sank 13 out of 20 basketball shots.
Who is the more accurate shooter?



Check your answers by turning to the Appendix.

Using Decimal Forms to Compare

Another way to compare ratios is to write them in their decimal forms.

Example

Luke hits safely 3 out of 8 times at bat. Jerritt hits safely 4 out of 9 times at bat. Who has the greater ratio of hits to times at bat?

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Solution

Step 1: Write the ratios in their decimal form. Round to the nearest thousandth.

$$\frac{\text{number of hits}}{\text{number of times at bat}}$$

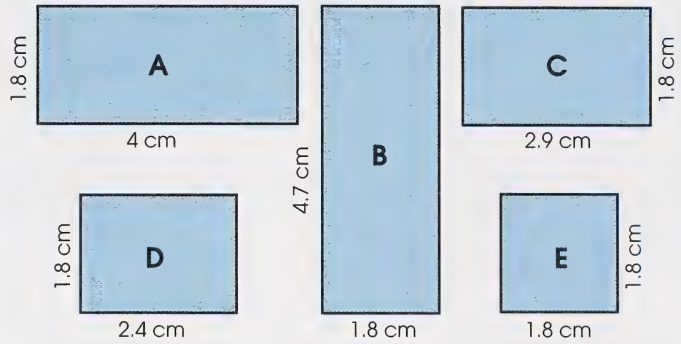
	Luke	Jerritt
	$\frac{3}{8} = 3 \div 8$	$\frac{4}{9} = 4 \div 9$
	$= 0.375$	$\doteq 0.444$

Step 2: Compare the decimal forms.

$$0.444 > 0.375$$

So, Jerritt has the greater ratio of hits to times at bat.

10. Use the following rectangles to answer 10.a., 10.b., and 10.c.



a. Which of the given rectangles appeals most to your eye?

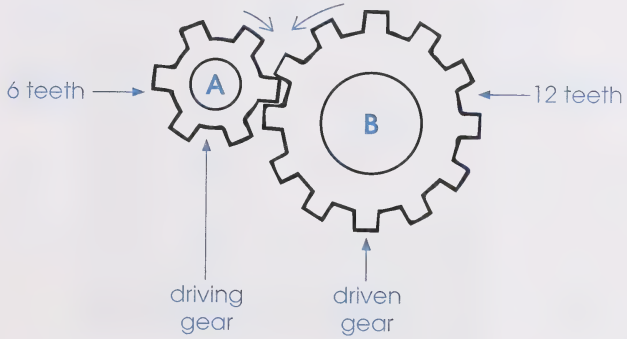
Use a calculator to answer question 10.b.

b. For each rectangle, find the ratio of its length to its width. Give each ratio in its decimal number form rounded to the nearest tenth. **Note:** Use the longer measurement as the length.

c. The ancient Greeks felt that the rectangles that were most appealing to the eye were those in which the ratio of the length to width was 1.618. They called this the **golden ratio**. They used rectangles with this ratio in many of their buildings, including the Parthenon at Athens.

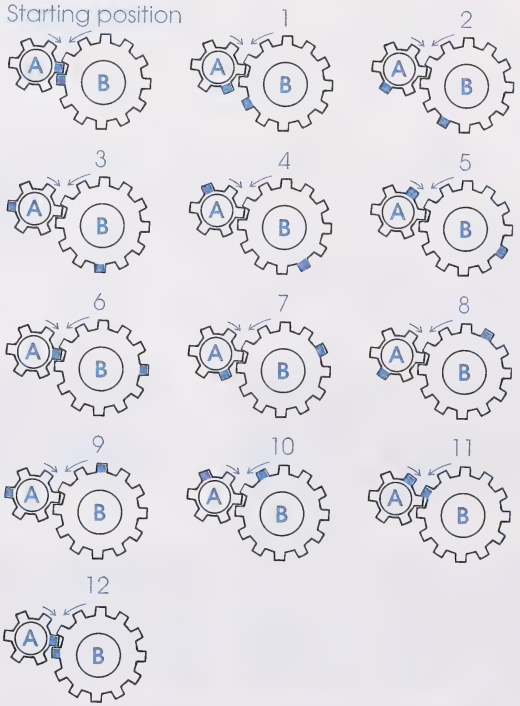
Which of the rectangles in this question has a ratio closest to the golden ratio? Was this rectangle the one you felt was most appealing?

11. a. Examine this diagram of a gear train. Notice that there are two gears. Gear A is the driving gear. Gear B is the driven gear.



What is the **gear ratio**? **Hint:** The gear ratio is the ratio of the number of teeth on the driving gear to the number of teeth on the driven gear.

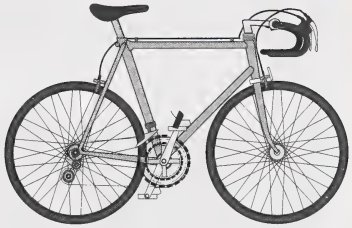
b. Look at the following diagram. Notice how the teeth of the two gears mesh. As Gear A moves, it drives Gear B. Also notice the direction each gear is moving. Gear A moves clockwise and Gear B moves counterclockwise.



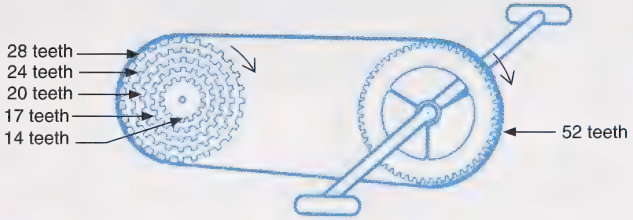
How many revolutions (complete turns) do Gear A and Gear B each make? **Hint:** Examine the position of the coloured teeth on each of the gears in the diagram.

c. What is the **turn ratio** of the gears in 11.b.? **Hint:** The turn ratio is the ratio of the number of turns of the driving gear to the number of turns of the driven gear.

12. On a five-speed bicycle, a chain connects the front gear and the back gear.



Examine this diagram of the gears on a five-speed bicycle. There is one gear on the pedal; it is the driving gear. There are five gears on the back wheel; they are the driven gears. The gears are attached by a chain and can be connected in different positions. Notice that the front and back gears move in the same direction.



Complete a chart like the one that follows. Use a calculator to find the gear ratios and turn ratios in decimal form. Round to the nearest thousandth.

Gear	Number of Teeth on Front Gear	Number of Teeth on Back Gear	Gear Ratio	Turn Ratio
1st	52	28		
2nd	52	24		
3rd	52	20		
4th	52	17		
5th	52	14		

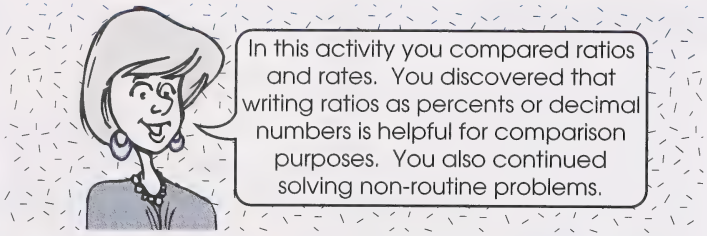
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following question.

13. Rico has an equal number of pennies, nickels, and quarters. If the value of the coins is $4.65, how many of each coin does he have?

Check your answer by turning to the Appendix.



Activity 2: Solving Ratio and Rate Problems

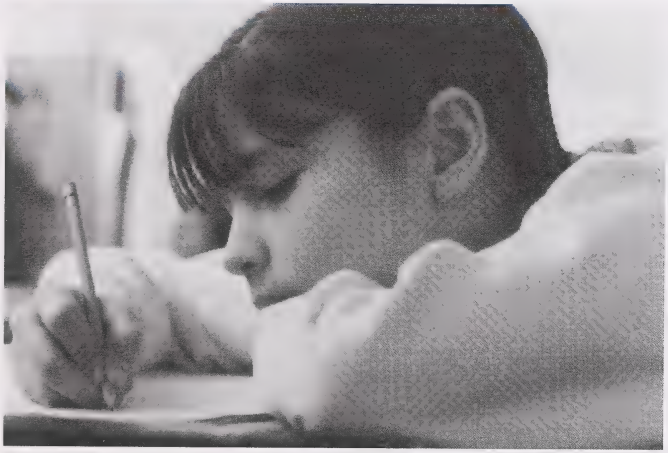


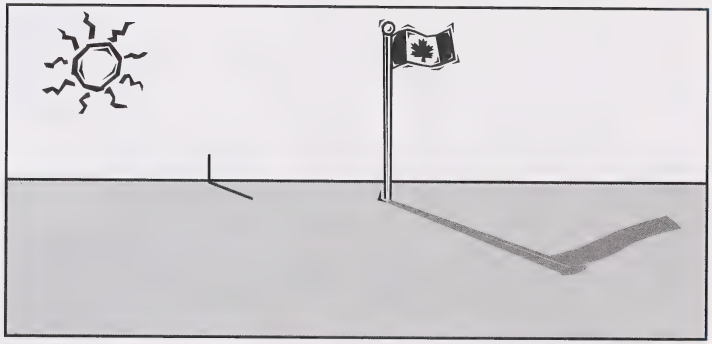
PHOTO SEARCH LTD.

One of the main goals of mathematics is to solve problems. In this activity you will use reasoning and your knowledge of proportions to solve ratio and rate problems.

Proportional reasoning allows you to solve ratio problems like the following.

Example 1

Malachi measures the shadow of a metre stick and the shadow of a flagpole at the same time of day. If the shadow of the metre stick is 3 m long and the shadow of the flagpole is 15 m long, how high is the flagpole?



Solution

Step 1: To solve the problem, write a proportion.

$$\frac{\text{height of object (m)}}{\text{length of shadow (m)}} \qquad \begin{matrix}\text{Metre Stick} \\ \frac{1}{3}\end{matrix} \quad = \quad \begin{matrix}\text{Flagpole} \\ \frac{}{15}\end{matrix}$$

Step 2: Find the missing term.

$$\frac{\text{height of object (m)}}{\text{length of shadow (m)}} \qquad \frac{1}{3} \overset{\times 5}{=} \frac{5}{15}$$

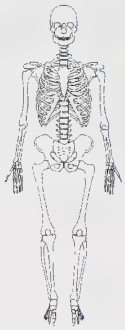
So, the flagpole is 5 m high.

1. A cookie recipe calls for 2 parts of flour to 1 part of sugar. If Tom uses 350 mL of flour, how much sugar does he need to make these cookies?

2. In a certain restaurant, the ratio of the number of cooks to the number of waiters and waitresses is 3 : 5. If there are 9 cooks, how many waiters and waitresses are there?

3. The ratio of canoes to sailboats on Thunder Lake is 7 to 2. If there are 21 canoes, how many sailboats are there?

4. The ratio of the number of bones in your head to the total number of bones in your body is about 1 to 7. If your body contains 206 bones, what is the approximate number of bones in your head?



Check your answers by turning to the Appendix.

Proportional reasoning allows you to solve ratio problems like the following.

Example 2

The ratio of the number of domestic cars to the number of foreign cars in a certain city is about 9 to 5. Estimate the number of foreign cars you could expect to find in a parking lot holding 280 cars.

Solution

If the ratio of the number of domestic cars to the number of foreign cars in a city is 9 to 5, the ratio of the number of foreign cars to the total number of cars in the city is 5 to 14.

Step 1: To solve the problem, write a proportion.

$$\frac{\text{number of foreign cars}}{\text{total number of cars}} \qquad \frac{5}{14} = \frac{}{280}$$

Step 2: Find the missing term.

$$\frac{\text{number of foreign cars}}{\text{total number of cars}} \qquad \frac{5}{14} \overset{\times 20}{=} \frac{100}{280}$$

So, you could expect to find about 100 foreign cars in the parking lot.

5. Sterling silver is an alloy of silver and copper in the ratio of 37 to 3. How many grams of silver are there in a sterling silver cup with a total mass of 400 g?

Check your answers by turning to the Appendix.

Proportional reasoning also allows you to solve rate problems like the following.

Example 3

Jessica is watching a storm. She sees lightning and 8 s later she hears thunder. How far away is the storm? **Hint:** Sound travels at the rate of 344 m/s.

Solution

Step 1: To solve the problem, write a proportion.

$$\frac{\text{distance (m)}}{\text{time (s)}} \qquad \frac{344}{1} = \frac{}{8}$$

Step 2: Find the missing term.

$$\frac{\text{distance (m)}}{\text{time (s)}} \qquad \frac{344}{1} \overset{\times 8}{=} \frac{2752}{8}$$

So, the storm is 2752 m away.

Example 4

Arthur is training for a 10-km race. He jogs 6 km daily and it takes him an average of 30 min to jog this distance. At this rate, how long will it take him to finish the 10-km race?

Solution

Step 1: Find Arthur's average rate of jogging speed.

$$\frac{\text{distance (km)}}{\text{time (min)}} \qquad \frac{6}{30} = \frac{0.2}{1}$$

Arthur jogs at a rate of 0.2 km/min.

Step 2: To solve the problem, write a proportion.

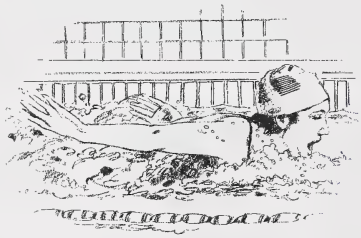
$$\frac{\text{distance (km)}}{\text{time (min)}} \qquad \frac{0.2}{1} = \frac{10}{\quad}$$

Step 3: Find the missing term.

$$\frac{\text{distance (km)}}{\text{time (min)}} \qquad \frac{0.2}{1} \overset{\times 50}{=} \frac{10}{50}$$

It will take Arthur 50 min to finish the 10-km race at this rate.

6. If Arlene swims at a rate of 2.5 m/s, how far can she swim in 50 s?



7. René, who works a 40-h week, is paid $450 per week. At the same hourly wage, how much will he receive if he works a 36-h week?

8. Chester travels 25 km in 2.5 h on his bike. At that rate, how long will it take him to travel 50 km?

Check your answers by turning to the Appendix.

Proportional reasoning allows you to solve currency problems like the following.

Example 5

This table shows the average cost of buying foreign currency in Canadian dollars for the second week in January 1995.

Exchange Rates	
U.S. dollar	1.43
Australian dollar	1.14
British pound	2.25
Dutch guilder	.8550
Fijian dollar	1.07
French franc	.2772
German mark	.9555
Italian lira	.000 919
Japanese yen	.015
Mexican peso	.3328
N.Z. dollar	.9468
Swiss franc	1.14
Greek drachma	.006 33

[1]

Using the exchange rates in the table, you can buy $1 U.S. for $1.43 Canadian. How much does it cost to buy $100 U.S.?

[1] Information supplied by Thomas Cook Group (Canada) Limited.

Solution

Step 1: Find the exchange rate.

$$\frac{\text{amount (Canadian dollars)}}{\text{amount (U.S. dollars)}} \qquad \frac{1.43}{1}$$

Step 2: To solve the problem, write a proportion. Find the missing term.

$$\frac{\text{amount (Canadian dollars)}}{\text{amount (U.S. dollars)}} \qquad \frac{1.43}{1} \overset{\times 100}{=} \frac{143}{100}$$

So, it costs $143 Canadian to buy $100 U.S.

9. Using the exchange rates from the table in Example 5, you can purchase 1000 Japanese yen for $15 Canadian. How many Japanese yen can be bought for $75 Canadian?

Check your answer by turning to the Appendix.

Use the Internet to find the exchange rates of different currencies today. **Hint:** The Klobas Currency Converter allows you to find the value of other currencies in Canadian dollars. This is its uniform resource locator (URL):

http://bin.gnn.com/cgi-bin/gnn/currency/

Now Try This

Use a problem-solving strategy to answer the following questions.

10. Geoff can run 5 m/s. Simon can run 7.5 m/s. If Geoff had a head start of 10 m, would he win a 100-m race?

11. Suki's record for the 100-m freestyle is 51.2 seconds. Her record for the 200-m freestyle is 1 minute 52.8 seconds. If Suki could swim the 200-m race at the same rate as the 100-m race, how much time would she cut from her record?

12. Nestor can type 30 words per minute. Leland can type 20 words per minute. Both boys are typing the same paragraph. If the paragraph contains 60 words, how much longer will the typing take Leland than Nestor?

Check your answers by turning to the Appendix.

Use a problem-solving strategy to answer the following question.

13. What number will have a remainder of 1 when divided by 2, 3, 4, or 6, but will have no remainder when divided by 7?

Check your answer by turning to the Appendix.

In this activity you used proportional reasoning to solve ratio and rate problems. You continued to solve non-routine problems.

Activity 3: Solving Percent Problems

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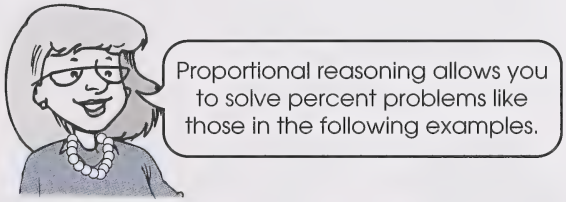
One of the main goals of mathematics is to solve problems. In this activity you will use reasoning and proportions to solve percent problems.



Example 1

The volleyball team sold $375 worth of chocolate bars. If the team kept 20% of the sales, how much did they keep?

Solution

Step 1: Write the percent as a ratio in simplest form.

$$20\% = \frac{20}{100}$$
$$= \frac{1}{5}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$$\frac{\text{amount kept (\$)}}{\text{amount sold (\$)}} \qquad \frac{1}{5} = \frac{75}{375}$$

$\times 75$ above the right fraction, $\times 75$ below the right fraction.

So, the team kept $75.

Example 2

Advance tickets to a rock concert are sold for $20 each. This is 80% of the price of the tickets at the door. How much will a ticket cost at the door?

Solution

Step 1: Write the percent as a ratio in simplest form:

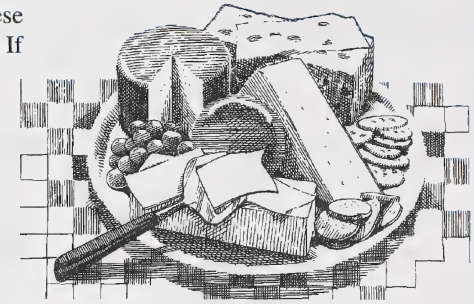
$$80\% = \frac{80}{100}$$
$$= \frac{4}{5}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$$\frac{\text{price of advance tickets (\$)}}{\text{cost of tickets at door (\$)}} \qquad \frac{4}{5} \overset{\times 5}{=} \frac{20}{25}$$

The price of a ticket at the door is $25.

1. A package of cheese has 23% milk fat. If the package holds 200 g of cheese, about how much milk fat is there in the package?



2. The Allans left their waiter a tip equal to 15% of the food bill. If the tip was $9, what was the food bill?

3. This year the profits of a local business were 130% of the profits last year. If the profits last year were $5200, what were the profits this year?

4. A survey found that at a certain school, 120 students (or about 80% of the students) held part-time jobs. How many students attend the school?

Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following questions.

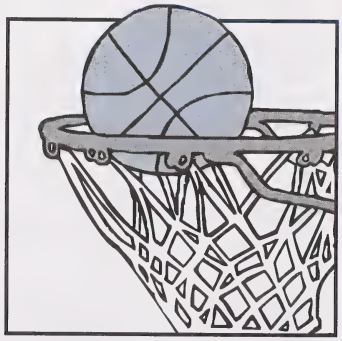
5. A farmer had a herd of cattle that he sold to four buyers. He sold 50% of the cattle to the first buyer, 25% of the cattle to the second buyer, 20% of the cattle to the third buyer, and 7 cattle to the fourth buyer. How many cattle did the farmer have?

6. Zoe's mark on a test was 80%. If there were 25 questions of equal value on the test, what was the ratio of the number of correct responses to the number of incorrect responses on Zoe's test?

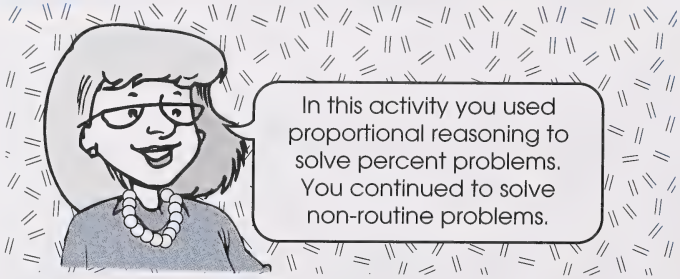
7. A bronze frame is made of 9 parts copper and 1 part tin.

a. What percent of the bronze frame is tin?
b. What percent of the bronze frame is copper?

8. Ernie made 8 out of 25 free throws, or 32% of his free throws. If he continues shooting, how many consecutive free throws will he have to make to raise his record to 50%?



Check your answers by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In the Extra Help for Section 1 you discovered an alternate method for simplifying ratios and rates. You may prefer this method for solving ratio problems.

Example 1

Darren saves $3 of every $4 he receives for his allowance. Last year he received $260 in allowances. How much did he save?

Solution

Step 1: To solve the problem, write a proportion.

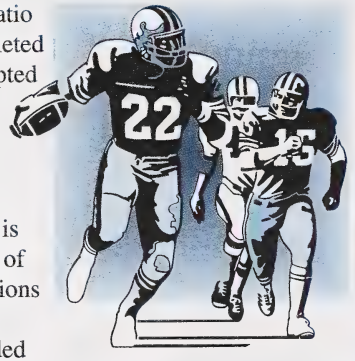
amount saved ($) to amount received ($)

$$3 : 4$$
$$= \quad : 260$$

Step 2: Find the missing term.

$$3 : 4$$
$$\times 65 \quad \times 65$$
$$= 195 : 260$$

Darren saved $195.



1. Denzil is a quarterback. His ratio of the number of passes completed to the number of passes attempted is 2 : 5. In a game Denzil completed 12 passes. How many passes did he attempt?

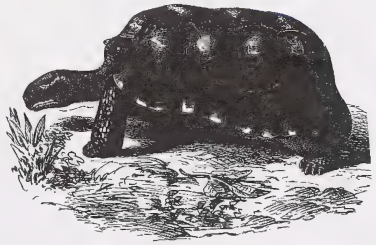
2. **a.** When a ten-speed bicycle is in first gear, 7 revolutions of the pedal make 10 revolutions of the wheel. How many pedal revolutions are needed for 120 wheel revolutions?

b. When a ten-speed bicycle is in tenth gear, 1 revolution of the pedal makes 3 revolutions of the wheel. How many pedal revolutions are needed for 120 wheel revolutions?

Check your answers by turning to the Appendix.

This alternate method for simplifying ratios and rates is also useful for solving rate problems.

Example 2



The very slow giant tortoise of Mauritius travels at a speed of 5 m/min. How long will it take the tortoise to travel 1 km?
Hint: 1 km = 1000 m

Solution

Step 1: To solve the problem, write a proportion.

distance travelled (m) per minute

$$\begin{aligned} &5 : 1 \\ =&1000 : \end{aligned}$$

Step 2: Find the missing term.

$$\begin{aligned} &5 : 1 \\ &\times 200 \quad \times 200 \\ =&1000 : 200 \end{aligned}$$

The tortoise will take 200 minutes to travel 1 km.

Step 3: Convert the time to hours.

60 min = 1 h

So, the tortoise will take 3 hours and 20 minutes to travel 1 km.

3. Your eye blinks about 25 times each minute. About how many times does your eye blink in an hour?
Hint: There are 60 min in 1 h.

4. Em's computer printer types 900 characters in 1 minute. How long will it take to print 7200 characters?

Check your answers by turning to the Appendix.

You can use this alternate method to solve percent problems.

Example 3

There are 200 students at Douglas Street School. Of these students, 13% are the youngest in their families. How many students in this school are the youngest in their families?

Solution

Step 1: To solve the problem, write a proportion.

number of youngest children to total number of students

$$13:100$$
$$= \square:200$$

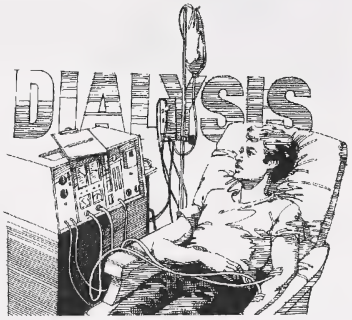
Step 2: Find the missing term.

$$13:100$$
$$\times 2 \quad \times 2$$
$$= 26:200$$

So, 26 of the students in the school are the youngest in their families.

Example 4

If 8% of a person's mass is blood, what is the mass of a person with 4 kg of blood?



Solution

Step 1: Write the percent as a ratio in simplest form.

$$8\% = \frac{8}{100}$$
$$= \frac{2}{25}$$
$$= 2:25$$

Step 2: To solve the problem, write a proportion.

mass of blood (kg) to mass of body (kg)

$$2:25$$
$$= 4:\square$$

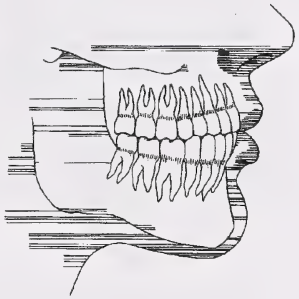
Step 3: Find the missing term.

$$2:25$$
$$\times 2 \quad \times 2$$
$$= 4:50$$

So, the person with 4 kg of blood has a mass of 50 kg.

5. In a certain school, 9% of the children were born outside Canada. If there are 200 children in the school, how many were born outside Canada?

6. An adult has 32 teeth. If 25% of the teeth are incisors, how many incisors are there?



7. The number of games the Flyers won is 300% of the number of games they lost. If the Flyers lost 4 games, how many games did they win?

Check your answers by turning to the Appendix.

Enrichment

You can use the decimal form of a percent to find a percent of a number.

Example 1

Andy scored 60% on a test with 25 questions of equal value. How many questions did Andy answer correctly?

Solution

Step 1: Change 60% to a decimal in simplest form.

$$60\% = \frac{60}{100}$$
$$= 0.60$$
$$= 0.6$$

Step 2: Find 60% of 25.

The word *of* means multiply.

$$60\% \text{ of } 25 = 0.6 \times 25$$
$$= 15$$

So, Andy answered 15 questions correctly.

1. There are 20 mL of milk fat in a 1-L carton of 2% milk. Find the amount of milk fat contained in 1000 mL of each type of dairy product in the given chart.

Type of Dairy Product	Milk Fat (%)	Amount of Milk Fat in 1000 mL
whipping cream	35	
coffee cream	18	
cereal cream	9	
partly skimmed milk	1	

2. Martinez uses solder to fix pipe connections. Solder is a mixture of tin and lead. In hard solder, 65% of the total mass is tin. Find the amount of tin in a 5-kg bar of hard solder.

3. The seating capacity of a football stadium is 10 800. At one game 75% of the seats were occupied. How many people were at the football game?

Check your answers by turning to the Appendix.

Many calculators have a [%] key. This key is helpful when you are finding a percent of a number.

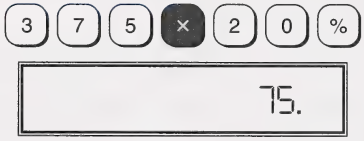
Example 2

A basketball team collected $375 in game admissions. If they kept 20% of this amount, how much did they keep for themselves?

Solution

Use a calculator to find the answer.

Enter the number to be multiplied first and the percent key last. It is not necessary to press the [=] key.



So, the team kept $75.

Note: Directions for using the [%] key on your calculator may differ. Check your owner's manual if these instructions do not work.

Use a calculator to answer questions 4 to 7.

4. The Kalchik family is planning a trip. They have to travel 1600 km. They want to travel 15% of this distance before stopping for lunch. How far will they travel before lunch?

5. A school librarian bought 120 new books. If 35% of the new books are adventure stories, how many adventure stories did the librarian buy?

6. The number of red balloons in a package is 400% the number of yellow balloons. If there are 15 yellow balloons in the package, how many red balloons are there?

7. Mrs. Ricci's monthly food expenses are 120% of Mrs. Morizama's monthly food expenses. If Mrs. Morizama spends $380 each month on food, how much does Mrs. Ricci spend?

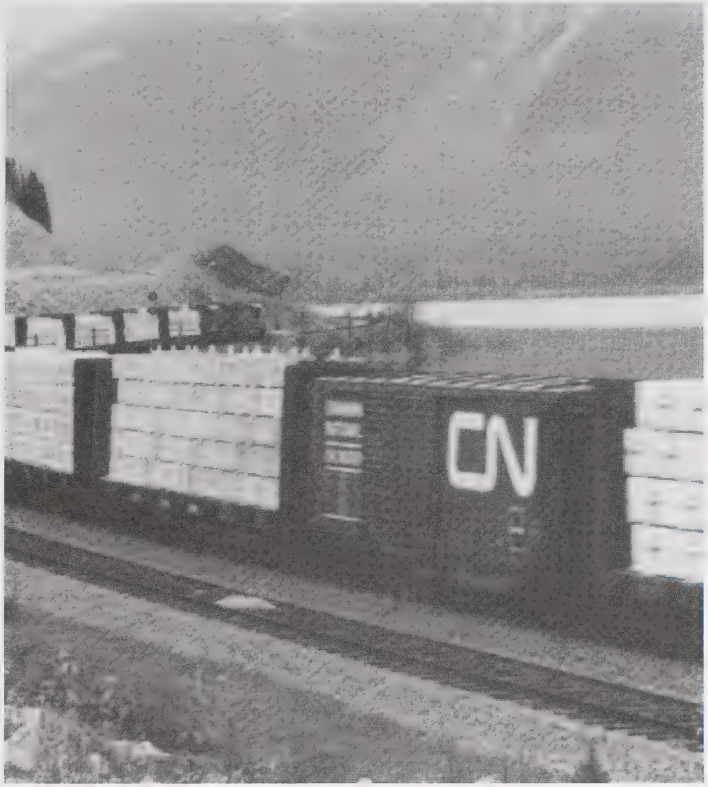
Check your answers by turning to the Appendix.

Did You Know?

The Percent Symbol

The use of percent in computing dates back to the Roman Empire; however, the percent symbol is the result of time-saving shortcuts used by Italian clerks in the fifteenth century. These clerks wrote the phrase *per cento* as *p cento* or *p c°*. Later, the phrase was abbreviated further, and was written as $P\overset{\circ}{/}$, then $\overset{\circ}{/}$, then $\frac{o}{o}$, and finally %.

Conclusion



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In this section you used proportional reasoning to solve ratio, rate, and percent problems.

You should now be able to solve problems like the following:

- If there are fifty cars in this train and the load limit is 118.5 tonnes per car, what is the load limit of this train?
- If ten cars of this train are left on a siding, what percent of the load is still left on the train?

Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 3.

Module Summary

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You explored ratios and proportions in this module. You distinguished between a ratio and a rate. You wrote ratios as percents. You changed percents less than 100% to fractions and decimals. You converted percents greater than 100% to whole numbers, mixed numbers, and decimal numbers. You used proportional reasoning to solve ratio, rate, and percent problems.

You should now be able to solve problems like the following:

- What is the ratio of the number of red bubble-gum balls to the number of blue bubble-gum balls in a bubble-gum machine?
- If you can purchase five bubble-gum balls for $1, what is the cost per bubble-gum ball?
- If the bubble-gum machine is 50% full, what fraction of the machine is full?

Final Module Assignment

Assignment Booklet

You are now ready to do the final module assignment.

APPENDIX

Glossary

Suggested Answers

Glossary

First term: (of a ratio) the first number in a ratio

Lowest terms: (of a ratio) a ratio in simplest form written with the lowest whole-number terms possible; also called **simplest form**

Percent: a special ratio with 100 as the second term

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Proportion: an equation that shows that two ratios are equivalent

Proportional ratios: equivalent ratios

Rate: a comparison that describes a unit relationship as well as a number relationship

Ratio: a comparison that describes a number relationship

Second term: (of a ratio) the second number in a ratio

Simplest form: (of a ratio) a ratio written in **lowest terms**

Suggested Answers

Section 1: Activity 1

1. a. The ratio of the number of parallelograms in Figure 1 to the number of parallelograms in Figure 2 is 1 to 6.

b. The ratio of the number of hexagons in Figure 1 to the number of hexagons in Figure 2 is 3 to 1.

c. The ratio of the number of hexagons in Figure 1 to the total number of hexagons in both figures is 3 to 4.

2. a. In Figure 1, the ratio of the number of parallelograms to the number of hexagons is 1 to 3.

b. In Figure 1, the ratio of the number of hexagons to the number of triangles is 3 to 2.

c. In Figure 1, the ratio of the number of parallelograms to the total number of pattern blocks is 1 to 6.

3. a. The ratio of the number of soccer balls to the number of footballs is 5 to 2.

The ratio of the number of soccer balls to the number of footballs is $5:2$.

The ratio of the number of soccer balls to the number of footballs is $\frac{5}{2}$.

b. The ratio of the number of basketballs to the total number of balls is 1 to 11.

The ratio of the number of basketballs is the total number of balls is $1:11$.

The ratio of the number of basketballs to the total number of balls is $\frac{1}{11}$.

4. a. The win-loss ratio is 41 to 10.
The win-loss ratio is 41:10.
The win-loss ratio is $\frac{41}{10}$.

b. The ratio of the number of wins to the total number of games played is 41 to 59.

The ratio of the number of wins to the total number of games played is 41:59.

The ratio of the number of wins to the total number of games played is $\frac{41}{59}$.

5. a. The first term of the ratio is 13.
b. The first term of the ratio refers to the number of boys.

6. a. The ratio of the number of hexagons to the number of parallelograms in the design is 3 to 12, or 1 to 4.

b. The ratio of the number of parallelograms to the total number of blocks in the design is 12 to 15, or 4 to 5.

7. a. $\frac{28}{16} \overset{\div 4}{=} \frac{7}{4}$

The ratio of the number of cats to the number of dogs is 7:4.

b. $\frac{10}{20} \overset{\div 10}{=} \frac{1}{2}$

The ratio of the number of rabbits to the number of birds is 1:2.

c. **Step 1:** Find the total number of animals.

$$28+16+10+20+26=100$$

Step 2: Write the ratio in lowest terms.

$$\frac{26}{100} \overset{\div 2}{=} \frac{13}{50}$$

The ratio of the number of fish to the total number of animals is 13:50.

8. a. **Step 1:** Find the total number of medals won by West Germany in the 1988 Winter Olympics.

$$2+4+2=8$$

Step 2: Write the ratio in lowest terms.

$$\frac{2}{8} \overset{\div 2}{=} \frac{1}{4}$$

The ratio of the number of gold medals won by West Germany to the total number of medals won by West Germany in the 1988 Winter Olympics is 1 : 4.

b. Step 1: Find the total number of medals won by Switzerland in the 1988 Winter Olympics.

$$5 + 5 + 5 = 15$$

Step 2: Write the ratio in lowest terms.

$$\frac{5}{15} \overset{\div 5}{=} \frac{1}{3}$$

The ratio of the number of gold medals won by Switzerland to the total number of medals won by Switzerland in the 1988 Winter Olympics is 1 : 3.

9. The ratio of the number of Canadian stamps to the number of American stamps Johann buys is $\frac{10}{3}, \frac{20}{6}, \frac{30}{9}, \frac{40}{12}, \ldots$.

10. The ratio of the number of light bulbs with defects to the total number of light bulbs Mikka inspects is $\frac{3}{100}, \frac{6}{200}, \frac{9}{300}, \frac{12}{400}, \ldots$.

Now Try This

11. a.

b.

6 is between 3 and 9.

c.

d.

12 is between 9 and 15.

12. One way to solve this problem is to make an organized list of the possible order of the animals:

- Bull, Bear, Rooster
- Bull, Rooster, Bear
- Bear, Bull, Rooster
- Bear, Rooster, Bull
- Rooster, Bear, Bull
- Rooster, Bull, Bear

Use the truth tables to check the possible orders. Read the statements each animal made in the problem. Then mark **T** or **F** in the truth table.

For example, if the animals were in the order of bull, bear, and rooster, the bull would not say "The bull is next to me" because the bull always tells the truth. The bear might say "I am the bull" because he sometimes tells the truth. The rooster would say "The rooster is next to me" because the rooster always lies.

Bull, Bear, Rooster	F	T	T	✗
Bull, Rooster, Bear	F	T	T	✗
Bear, Bull, Rooster	T	F	T	✗
Bear, Rooster, Bull	T	T	T	✓
Rooster, Bear, Bull	T	T	F	✗
Rooster, Bull, Bear	F	F	T	✗

So, the animals are arranged in this order: Bear, Rooster, Bull.

Section 1: Activity 2

1. a. The ratio of Franz's height to his father's height is 199 to 175.

b. The ratio of Franz's height to his mother's height is 199 to 167.

2. a. The ratio of Emma's age to Mavis's age is 12 to 13.
b. The ratio of Mavis's age to Emma's age is 13 to 12.

3. a. The ratio of the amount of red paint to the amount of yellow paint is 1 to 2.

b. The ratio of the amount of red paint to the amount of yellow paint is $1:2$.

c. The ratio of the amount of red paint to the amount of yellow paint is $\frac{1}{2}$.

4. a. The ratio of the height of the tree to the length of its shadow is 10 to 7.

b. The ratio of the height of the tree to the length of its shadow is $10:7$.

c. The ratio of the height of the tree to the length of its shadow is $\frac{10}{7}$.

5. $\dfrac{\text{amount in larger tube (mL)}}{\text{amount in smaller tube (mL)}}$

$\dfrac{200}{100} \overset{\div 100}{=} \dfrac{2}{1}$

The ratio of the amount of toothpaste in the larger tube to the amount of toothpaste in the smaller tube is $2:1$.

6. a. $\dfrac{\text{mass of gravel (kg)}}{\text{mass of sand (kg)}}$

$\dfrac{300}{450} \overset{\div 150}{=} \dfrac{2}{3}$

The ratio of the mass of gravel to the mass of sand is $2:3$.

b. $\dfrac{\text{mass of gravel (kg)}}{\text{mass of cement (kg)}}$

$\dfrac{300}{100} \overset{\div 100}{=} \dfrac{3}{1}$

The ratio of the mass of gravel to the mass of cement is $3:1$.

7. $\dfrac{\text{time slept Monday night (h)}}{\text{time slept Tuesday night (h)}}$

$\dfrac{10}{8} \overset{\div 2}{=} \dfrac{5}{4}$

The ratio of the time Brian slept Monday night to the time he slept Tuesday night is $5:4$.

8. Step 1: Write the measurements in same unit.

$2 \text{ kg} = 2000 \text{ g}$

Step 2: Write the ratio in lowest terms.

$\dfrac{\text{mass of pumpkin (g)}}{\text{mass of zucchini (g)}}$

$\dfrac{2000}{250} \overset{\div 250}{=} \dfrac{8}{1}$

The ratio of the mass of the pumpkin to the mass of the zucchini is $8:1$.

9. Step 1: Write the measurements in the same unit.

$5 \text{ L} = 5000 \text{ mL}$

Step 2: Write the ratio in lowest terms.

$\dfrac{\text{amount of blood donated (mL)}}{\text{amount of blood in the body (mL)}}$

$\dfrac{450}{5000} \overset{\div 50}{=} \dfrac{9}{100}$

The ratio of the amount of blood donated to the amount of blood in the entire body is $9:100$.

10. The ratio of the amount of concentrate to the amount of water is $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots$.

11. The ratio of the recommended number of students to the number of teachers is $\frac{20}{1}, \frac{40}{2}, \frac{60}{3}, \frac{80}{4}, \ldots$.

Now Try This

12. You may find it helpful to use diagrams or objects to solve this problem.

a. Fill the 5-L pail with water. Gradually pour the water from the 5-L pail into the 3-L pail until the 3-L pail is full.

There is exactly 2 L left in the 5-L pail.

b. **Step 1:** Fill the 3-L pail with water. Pour the water from the 3-L pail into the 5-L pail.

Step 2: Fill the 3-L pail with water. Gradually pour the water from the 3-L pail into the 5-L pail until the 5-L pail is full.

There is exactly 1 L left in the 3-L pail.

c. **Step 1:** Fill the 3-L pail with water. Pour the water from the 3-L pail into the 5-L pail.

Step 2: Fill the 3-L pail with water. Gradually pour the water from the 3-L pail into the 5-L pail until the 5-L pail is full. There will be 1 L left in the 3-L pail.

Step 3: Empty the water from the 5-L pail. Pour the water from the 3-L pail into the 5-L pail. There is now 1 L of water in the 5-L pail.

Step 4: Fill the 3-L pail with water. Pour the water from the 3-L pail into the 5-L pail.

There are now exactly 4 L of water in the 5-L pail.

Section 1: Activity 3

1. **a.** The rate of cost is \$3.99 per 12 doughnuts.
b. The rate of cost is \$3.99 : 12 doughnuts.
c. The rate of cost is $\frac{\$3.99}{12 \text{ doughnuts}}$.
d. The rate of cost is \$3.99/12 doughnuts.

2. **a.** The rate of sales is 7 cars per 10 days.
b. The rate of sales is 7 cars : 10 days.
c. The rate of sales is $\frac{7 \text{ cars}}{10 \text{ days}}$.
d. The rate of sales is 7 cars/10 days.

3. **a.** The rate of production is 2 afghans per 5 weeks.
b. The rate of production is 2 afghans : 5 weeks.
c. The rate of production is $\frac{2 \text{ afghans}}{5 \text{ weeks}}$.
d. The rate of production is 2 afghans/5 weeks.

4. **a.** The rate of stretch is 1 cm per 25 g.
b. The rate of stretch is 1 cm : 25 g.
c. The rate of stretch is $\frac{1 \text{ cm}}{25 \text{ g}}$.
d. The rate of stretch is 1 cm/25 g.

5. a. The rate of coverage is 4 L per 25 m^2.

b. The rate of coverage is 4 L : 25 m^2.

c. The rate of coverage is $\frac{4\text{ L}}{25\text{ m}^2}$.

d. The rate of coverage is 4 L/25 m^2

6. a.

$$\frac{\text{number of words}}{\text{amount of time (min)}} \qquad \frac{420}{4} \overset{\div 4}{=} \frac{105}{1}$$

Bob's typing rate is 105 words per minute.

b.

$$\frac{\text{number of beats}}{\text{amount of time (min)}} \qquad \frac{300}{4} \overset{\div 4}{=} \frac{75}{1}$$

Evan's heart rate is 75 beats per minute.

c.

$$\frac{\text{amount of stretch (cm)}}{\text{amount of mass (g)}} \qquad \frac{2}{100} \overset{\div 100}{=} \frac{0.02}{1}$$

The rate of stretch is 0.02 cm per gram.

d.

$$\frac{\text{amount in Canadian dollars}}{\text{amount in British pounds}} \qquad \frac{225}{100} \overset{\div 100}{=} \frac{2.25}{1}$$

The exchange rate is about $2.25 Canadian per British pound.

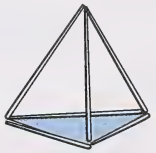
7. Rita's rate of pay is \$8.20/h, \$16.40/2 h, \$24.60/3 h, \$32.80/4 h,... .

8. Arlene's rate of speed is 2.5 m/s, 5 m/2 s, 7.5 m/3 s, 10 m/4 s,... .

Now Try This

9. To solve this problem you need to change your point of view. It is impossible to form 4 congruent triangles by arranging the toothpicks flat on a table.

The problem can be solved by arranging the toothpicks to form a three-dimensional pyramid.



Section 1: Follow-up Activities

Extra Help

1. a. number of student tickets sold to the number of adult tickets sold

$$20:15$$
$$\div 5 \quad \div 5$$
$$=4:3$$

The ratio of the number of student tickets sold to the number of adult tickets sold is $4:3$.

b. price of CDs to price of cassettes

$$16:10$$
$$\div 2 \quad \div 2$$
$$=8:5$$

The ratio of the price of CDs to the price of cassettes is $8:5$.

2. a. amount of rain (mm) per amount of time (h)

$$42:10$$
$$\div 10 \quad \div 10$$
$$=4.2:1$$

The rate of rainfall is 4.2 mm/h.

b. amount in Canadian dollars per amount in U.S. dollars

$$28:20$$
$$\div 20 \quad \div 20$$
$$=1.40:1$$

The exchange rate is \$1.40 Canadian/U.S. dollar.

Enrichment

1. $\dfrac{\text{amount of fuel consumed (L)}}{\text{distance travelled (km)}}$

$$\frac{72}{800} \overset{\div 8}{=} \frac{9}{100} \quad (\div 8 \text{ below})$$

The fuel consumption rate is 9 L/100 km.

2.

$$\frac{\text{cost (\$)}}{\text{mass (g)}} \qquad \frac{6.50}{500} = \frac{1.30}{100} \qquad \div 5 \text{ (top)}, \ \div 5 \text{ (bottom)}$$

The unit cost rate is \$1.30/100 g.

3. **Step 1:** Write the measurement in millilitres.

$$1 \text{ L} = 1000 \text{ mL}$$

Step 2: Simplify the rate.

$$\frac{\text{cost (\$)}}{\text{volume (mL)}} \qquad \frac{\$1.29}{1000} \doteq \frac{\$0.13}{100} \qquad \div 10 \text{ (top)}, \ \div 10 \text{ (bottom)}$$

The unit cost rate is about \$0.13/100 mL.

4.

$$\frac{\text{number of divorces}}{\text{total number of people}} \qquad \frac{61\,980}{25\,360\,000} \doteq \frac{2.44}{1000} \qquad \div 25\,360 \text{ (top)}, \ \div 25\,360 \text{ (bottom)}$$

The divorce rate in Canada in 1985 was about 2.44 divorces per 1000 population.

Section 2: Activity 1

1. a. $\frac{20}{100} = \frac{1}{5}$

The number of raisins is $\frac{1}{5}$ of the number of bran flakes.

b. $\frac{20}{100} = 0.20$

$= 0.2$

The number of raisins is 0.2 of the number of bran flakes.

c. $\frac{20}{100} = 20\%$

The number of raisins is 20% of the number of bran flakes.

2. a. $\frac{5}{100} = \frac{1}{20}$

The length of a grasshopper is $\frac{1}{20}$ of the distance the grasshopper can jump.

b. $\frac{5}{100} = 0.05$

The length of a grasshopper's body is 0.05 of the distance the grasshopper can jump.

c. $\frac{5}{100} = 5\%$

The length of a grasshopper's body is 5% of the distance the grasshopper can jump.

3. a. $\frac{2}{100} = \frac{1}{50}$

The mass of an ant's own body is $\frac{1}{50}$ of the mass it can lift.

b. $\frac{2}{100} = 0.02$

The mass of an ant's own body is 0.02 of the mass it can lift.

c. $\frac{2}{100} = 2\%$

The mass of an ant's own body is 2% of the mass it can lift.

4. a. $\frac{35}{100} = \frac{7}{20}$

The number of baskets is $\frac{7}{20}$ of the number of attempts.

b. $\frac{35}{100} = 0.35$

The number of baskets is 0.35 of the number of attempts.

c. $\frac{35}{100} = 35\%$

The number of baskets is 35% of the number of attempts.

5. a. The number of students present is $\frac{3}{4}$ of the number of students in the class.

b. $\frac{3}{4} = \frac{75}{100}$
$= 0.75$

The number of students present is 0.75 of the number of students in the class.

c. $\frac{3}{4} = \frac{75}{100}$
$= 75\%$

The number of students present is 75% of the number of students in the class.

6. a. The number of pages Bruce read is $\frac{4}{5}$ of the number of pages Tony read.

b. $\frac{4}{5} = \frac{8}{10}$
$= 0.8$

The number of pages Bruce read is 0.8 of the number of pages Tony read.

c. $\frac{4}{5} = \frac{80}{100}$

$= 80\%$

The number of pages Bruce read is 80% of the number of pages Tony read.

7. a. The length of the nail is $\frac{3}{5}$ of the length of the pencil.

b. $\frac{3}{5} = \frac{6}{10}$

$= 0.6$

The length of the nail is 0.6 of the length of the pencil.

c. $\frac{3}{5} = \frac{60}{100}$

$= 60\%$

The length of the nail is 60% of the length of the pencil.

8. $0.75 = \frac{75}{100}$

$= 75\%$

The price of advance tickets is 75% of the price of tickets at the door.

9. $0.390 = 0.39$

$= \frac{39}{100}$

$= 39\%$

In 1914, Ty Cobb hit safely 39% of the times he was up to bat.

10. $\frac{3}{4} = \frac{75}{100}$

$= 75\%$

In Wasylena's neighbourhood, 75% of the pets are dogs.

11. $\frac{1}{5} = \frac{20}{100}$

$= 20\%$

Mr. Sabban sowed 20% of his fields in barley.

12. a. $20\% = \frac{20}{100}$

$= 0.20$

$= 0.2$

The students kept 0.2 of the sales.

b. $20\% = \frac{20}{100}$

$= \frac{1}{5}$

The students kept $\frac{1}{5}$ of the sales.

13. a. $60\% = \frac{60}{100}$

$= 0.60$

$= 0.6$

The amount of tin is 0.6 of the amount of lead.

b. $60\% = \frac{60}{100}$

$= \frac{3}{5}$

The amount of tin is $\frac{3}{5}$ of the amount of lead.

14. a. $35\% = \frac{35}{100}$

$= 0.35$

Darryl made 0.35 of the shots he attempted.

b. $35\% = \frac{35}{100}$

$= \frac{7}{20}$

Darryl made $\frac{7}{20}$ of the shots he attempted.

Now Try This

15. Answers may vary. To solve this problem, you may use the guess, check, and revise method. One example is given here.

Guess 1

	Number	Value in Cents
quarters	3	75
dimes	2	20
nickels	4	20
pennies	3	3
Total	12	118

← too high

$118¢ = \$1.18$

$1.18 > 1.13$

Guess 2

	Number	Value in Cents
quarters	3	75
dimes	1	10
nickels	5	25
pennies	3	3
Total	12	113

← correct

$113¢ = \$1.13$

Erika has 3 quarters, 1 dime, 5 nickels, and 3 pennies.

Section 2: Activity 2

1. a. $\dfrac{\text{amount of cranberry juice (mL)}}{\text{amount of apple juice (mL)}} \quad \dfrac{375}{375} = \dfrac{1}{1}$

The ratio of the amount of cranberry juice to the amount of apple juice is 1 to 1.

b. $\dfrac{\text{amount of cranberry juice (mL)}}{\text{amount of apple juice (mL)}} \quad \dfrac{375}{375} = \dfrac{1}{1} = 1$

The amount of cranberry juice is 1 times the amount of apple juice.

c. $\dfrac{\text{amount of cranberry juice (mL)}}{\text{amount of apple juice (mL)}} \quad \dfrac{375}{375} = \dfrac{1}{1} = \dfrac{100}{100} = 100\%$

The amount of cranberry juice is 100% of the amount of apple juice.

2. a. $\dfrac{\text{number of cracked eggs}}{\text{total number of eggs}} \quad \dfrac{12}{12} = \dfrac{1}{1}$

The ratio of the number of cracked eggs to the total number of eggs is 1 to 1.

b. $\dfrac{\text{number of cracked eggs}}{\text{total number of eggs}} \quad \dfrac{12}{12} = \dfrac{1}{1} = 1$

The number of cracked eggs is 1 times the total number of eggs.

c. $\dfrac{\text{number of cracked eggs}}{\text{total number of eggs}} \quad \dfrac{12}{12} = \dfrac{1}{1} = \dfrac{100}{100} = 100\%$

The number of cracked eggs is 100% of the total number of eggs.

3. $100\% - 30\% = 70\%$

Ramon lost 70% of the ski races he entered.

4. $100\% - 5\% = 95\%$

In the group, 95% of the children are right-handed.

5. $100\% - 20\% = 80\%$

In the group, 80% of the men do not wear glasses.

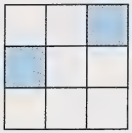
6.
$$\begin{array}{r} 30\% \\ 25\% \\ +\ 15\% \\ \hline 70\% \end{array} \qquad 100\% - 70\% = 30\%$$

In the box, 30% of the stars are yellow.

Now Try This

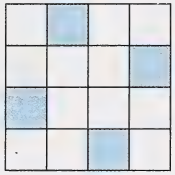
7. You can find and apply a pattern.

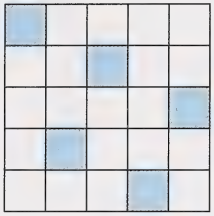
In a 2×2 square, 1 small square can be shaded.



In a 3×3 square, 2 small squares can be shaded.

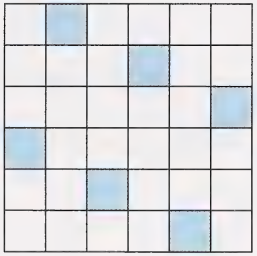
In a 4×4 square, 4 small squares can be shaded.





In a 5×5 square, 5 small squares can be shaded.

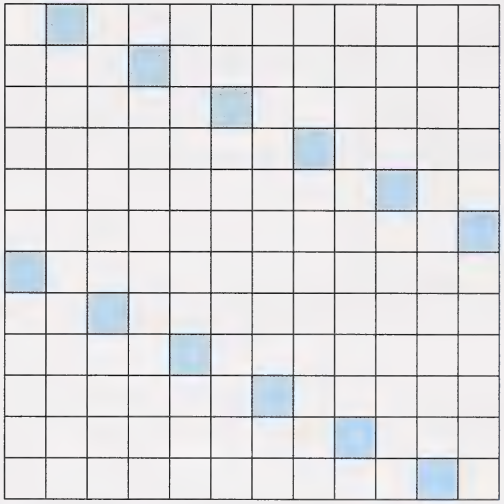
In a 6×6 square, 6 small squares can be shaded.



The pattern can be shown in a table.

Size of Square	Number of Shaded Squares
2×2	1
3×3	2
4×4	4
5×5	5
6×6	6

If the square is greater than 3×3, the number of shaded squares is equal to the number of rows or the number of columns. So, in a 12×12 square, 12 squares can be shaded.



Section 2: Activity 3

1. a.

$$\frac{\text{number of eggs laid this year}}{\text{number of eggs laid last year}} \qquad \frac{156}{52} = \frac{3}{1}$$

The ratio of the number of eggs laid this year to the number laid last year is 3 to 1.

b.

$$\frac{\text{number of eggs laid this year}}{\text{number of eggs laid last year}} \qquad \frac{156}{52} = \frac{3}{1}$$
$$= 3$$

The number of eggs laid this year is 3 times the number of eggs laid last year.

c.

$$\frac{\text{number of eggs laid this year}}{\text{number of eggs laid last year}} \qquad \frac{156}{52} = \frac{3}{1}$$
$$= \frac{300}{100}$$
$$= 300\%$$

The number of eggs laid this year is 300% of the number of eggs laid last year.

2. a.

$$\frac{\text{height of jump on the Moon (m)}}{\text{height of jump on Earth (m)}} \qquad \frac{12}{2} = \frac{6}{1}$$

The ratio of the height of the jump on the Moon to the height of the jump on Earth is 6 : 1.

b.

$$\frac{\text{height of jump on the Moon (m)}}{\text{height of jump on Earth (m)}} \qquad \frac{12}{2} = \frac{6}{1}$$
$$= 6 \div 1$$
$$= 6$$

The height of the jump on the Moon is 6 times the height of the jump on Earth.

c.

$$\frac{\text{height of jump on the Moon (m)}}{\text{height of jump on Earth (m)}} \qquad \frac{12}{2} = \frac{6}{1}$$
$$= \frac{600}{100}$$
$$= 600\%$$

The height of the jump on the Moon is 600% of the height of the jump on Earth.

3. a.

$$\frac{\text{height today (cm)}}{\text{height last year (cm)}} \qquad \frac{75}{60} = \frac{25}{20}$$
$$= \frac{5}{4}$$

The ratio of Bobby's height today to Bobby's height last year is 5 to 4.

b. $\frac{\text{height today (cm)}}{\text{height last year (cm)}}$

$$\frac{75}{60} = \frac{5}{4}$$
$$= 1\frac{1}{4}$$

Bobby's height today is $1\frac{1}{4}$ times his height last year.

c. $\frac{\text{height today (cm)}}{\text{height last year (cm)}}$

$$\frac{75}{60} = \frac{5}{4}$$
$$= 5 \div 4$$
$$= 1.25$$

Bobby's height today is 1.25 times his height last year.

d. $\frac{\text{height today (cm)}}{\text{height last year (cm)}}$

$$\frac{75}{60} = \frac{5}{4}$$
$$= \frac{125}{100}$$
$$= 125\%$$

Bobby's height today is 125% of his height last year.

4. a. $\frac{\text{number of canoes}}{\text{number of sailboats}}$

$$\frac{24}{15} = \frac{8}{5}$$

The ratio of the number of canoes on the lake to the number of sailboats is 8 to 5.

b. $\frac{\text{number of canoes}}{\text{number of sailboats}}$

$$\frac{24}{15} = \frac{8}{5}$$
$$= 1\frac{3}{5}$$

The number of canoes on the lake is $1\frac{3}{5}$ times the number of sailboats.

c. $\frac{\text{number of canoes}}{\text{number of sailboats}}$

$$\frac{24}{15} = \frac{8}{5}$$
$$= 1.6$$

The number of canoes on the lake is 1.6 times the number of sailboats.

d. $\frac{\text{number of canoes}}{\text{number of sailboats}}$

$$\frac{24}{15} = \frac{8}{5}$$
$$= \frac{160}{100}$$
$$= 160\%$$

The ratio of the number of canoes on the lake is 160% of the number of sailboats.

5. $6 = \frac{6}{1}$

$$= \frac{600}{100}$$
$$= 600\%$$

Some jet planes travel at a speed that is 600% of the flying speed of the spine-tail swift.

6. $2 = \frac{2}{1}$

$= \frac{200}{100}$

$= 200\%$

A supersonic jet can fly at a speed that is 200% of the speed of sound.

7. $1.56 = \frac{1.56}{1}$

$= \frac{156}{100}$

$= 156\%$

The mass of milk is 156% of the mass of gasoline.

8. $1\,000\,000 = \frac{1\,000\,000}{1}$

$= \frac{100\,000\,000}{100}$

$= 100\,000\,000\%$

The mass of an ostrich egg is 100 000 000% of the mass of a hummingbird egg.

9. $7.5 = \frac{7.5}{1}$

$= \frac{750}{100}$

$= 750\%$

The diameter of the wire to make coat hangers is 750% of the diameter of a human hair.

10. a. $700\% = \frac{700}{100}$

$= 700 \div 100$

$= 7$

The size of Earth is 7 times the size of Mars.

b. $1600\% = \frac{1600}{100}$

$= 1600 \div 100$

$= 16$

The size of Earth is 16 times the size of Mercury.

11. $167\,000\% = \frac{167\,000}{100}$

$= 1670$

The volume of steam is 1670 times the volume of water.

12. a. $150\% = \frac{150}{100}$

$= 1\frac{50}{100}$

$= 1\frac{1}{2}$

The average life span of a domestic sheep is $1\frac{1}{2}$ times the life span of a domestic goat.

b. $150\% = \frac{150}{100}$

$= 1.50$

$= 1.5$

The average life span of a domestic sheep is 1.5 times the life span of a domestic goat.

13. a. $240\% = \frac{240}{100}$

$= 2\frac{40}{100}$

$= 2\frac{2}{5}$

The average life span of a domestic cat is $2\frac{2}{5}$ times the life span of a domestic rabbit.

b. $240\% = \frac{240}{100}$

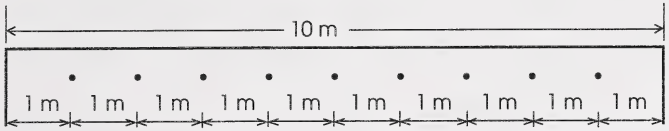
$= 2.40$

$= 2.4$

The average life span of a domestic cat is 2.4 times the life span of a domestic rabbit.

Now Try This

14. To solve this problem, you may use objects or make a diagram.



If the hooks are placed every metre, 9 hooks are needed.

Section 2: Follow-up Activities

Extra Help

1. 1% means 1 out of 100. In the video, 100 pennies were shown. Of the 100 pennies, 1 penny was turned over. This was 1% of the pennies.

2. 250% means 100% + 100% + 50%. In the video, an example of an elephant was used to illustrate 250%. The video explained that 2.5 elephants were equal to 250% of an elephant.

Enrichment

1.

Number	Percent
0.60	60
0.35	35
0.08	8
2	200
6.2	620

2.

Percent	Number
50	0.5
8	0.08
62	0.62
400	4
375	3.75

Section 3: Activity 1

1.

	Photo A	Photo B
$\frac{\text{length (mm)}}{\text{width (mm)}}$	$\frac{40}{32} = \frac{5}{4}$	$\frac{50}{40} = \frac{5}{4}$

$\frac{5}{4} = \frac{5}{4}$

So, the ratio of the lengths to the widths of the photographs are equivalent.

2.

	Cylinder A	Cylinder B
$\frac{\text{height (cm)}}{\text{diameter (cm)}}$	$\frac{2}{3}$	$\frac{3}{2}$

$\frac{2}{3} \neq \frac{3}{2}$

So, the ratios of the heights to the diameters of the cylinders are not equivalent.

3.

	Dino	Frank
$\frac{\text{distance (km)}}{\text{time (h)}}$	$\frac{830}{9} \doteq \frac{92.2}{1}$	$\frac{640}{7} \doteq \frac{91.4}{1}$

$\frac{92.2}{1} > \frac{91.4}{1}$

So, Dino travelled at the greater speed.

4.

	Athlete 1	Athlete 2
$\frac{\text{distance (m)}}{\text{speed (s)}}$	$\frac{100}{9.9} = \frac{10.1}{1}$	$\frac{200}{19.8} = \frac{10.1}{1}$

$\frac{10.1}{1} = \frac{10.1}{1}$

So, the athletes are running at the same speed.

5.

	2 Servings	4 Servings	6 Servings
$\frac{\text{rice (mL)}}{\text{water (mL)}}$	$\frac{125}{300} = \frac{5}{12}$	$\frac{175}{375} = \frac{7}{15}$	$\frac{250}{500} = \frac{1}{2}$
	$\doteq 0.41$	$= 0.46$	$= 0.5$

$0.41 \neq 0.46 \neq 0.5$

So, the ratio of the amount of rice to the amount of water is not proportional for 2, 4, and 6 servings.

6. a.

	Dressing	Situation A
$\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$	$\frac{2}{1}$	$\frac{60}{30} = \frac{2}{1}$

$\frac{2}{1} = \frac{2}{1}$

Yes, this could be the amount of oil and vinegar in the salad dressing.

b.

	Dressing	Situation B
$\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$	$\frac{2}{1}$	$\frac{30}{60} = \frac{1}{2}$

$\frac{2}{1} \neq \frac{1}{2}$

No, this could not be the amount of oil and vinegar in the salad dressing.

c.

	Dressing	Situation C
$\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$	$\frac{2}{1}$	$\frac{90}{45} = \frac{2}{1}$

$\frac{2}{1} = \frac{2}{1}$

Yes, this could be the amount of oil and vinegar in the salad dressing.

d.

$\frac{\text{oil (mL)}}{\text{vinegar (mL)}}$	Dressing	Situation D
	$\frac{2}{1}$	$\frac{45}{90} = \frac{1}{2}$

The ratios are not proportional.

No, this could not be the amount of oil and vinegar in the salad dressing.

7.

$\frac{\text{cost (\$)}}{\text{mass (kg)}}$	Rate 1	Rate 2	Rate 3
	$\frac{8.60}{2} = \frac{4.30}{1}$	$\frac{17.20}{4} = \frac{4.30}{1}$	$\frac{25.80}{6} = \frac{4.30}{1}$

Yes, the rates are proportional.

8.

$\frac{\text{score}}{\text{total}}$	Test 1	Test 2
	$\frac{17}{25} = \frac{68}{100}$ $= 68\%$	$\frac{14}{20} = \frac{70}{100}$ $= 70\%$

$70\% > 68\%$

So, Kara did better on the second test.

9.

$\frac{\text{shots}}{\text{attempts}}$	Matt	Jon
	$\frac{7}{10} = \frac{70}{100}$ $= 70\%$	$\frac{13}{20} = \frac{65}{100}$ $= 65\%$

$70\% > 65\%$

So, Matt is the more accurate shooter.

10. a. Answers will vary.

b.

$\frac{\text{length (cm)}}{\text{width (cm)}}$	A	B	C
	$\frac{4}{1.8} \div 2.2$	$\frac{4.7}{1.8} \div 2.6$	$\frac{2.9}{1.8} \div 1.6$

D	E
$\frac{2.4}{1.8} \div 1.3$	$\frac{1.8}{1.8} = 1$

c. Rectangle C has a ratio closest to the golden ratio. Did you find Rectangle C the most appealing?

11. a. $\frac{\text{number of teeth on driving gear}}{\text{number of teeth on driven wheel}}$ $\frac{6}{12} = \frac{1}{2}$

The gear ratio is 1 : 2.

b. Gear A makes 2 complete revolutions.
Gear B makes 1 complete revolution.

c. $\dfrac{\text{number of turns of driving gear}}{\text{number of turns of driven gear}} \quad \dfrac{2}{1}$

So, the turn ratio is $2:1$.

12.

Gear	Number of Teeth on Front Gear	Number of Teeth on Back Gear	Gear Ratio	Turn Ratio
1st	52	28	1.857	0.538
2nd	52	24	2.167	0.462
3rd	52	20	2.600	0.385
4th	52	17	3.059	0.327
5th	52	14	3.714	0.269

Now Try This

13. You may use the guess, check, and revise strategy to solve this problem. One example is given here.

Guess 1: Rico has 10 pennies, 10 nickels, and 10 quarters.

$$\begin{aligned} 10 \times 0.01 &= 0.10 \\ 10 \times 0.05 &= 0.50 \\ 10 \times 0.25 &= \underline{2.50} \\ &\quad 3.10 \end{aligned}$$

$3.10 < 4.65$

This total is too low.

Guess 2: Rico has 15 pennies, 15 nickels, and 15 quarters.

$$\begin{aligned} 15 \times 0.01 &= 0.15 \\ 15 \times 0.05 &= 0.75 \\ 15 \times 0.25 &= \underline{3.75} \\ &\quad 4.65 \end{aligned}$$

This total is correct.

So, Rico has 15 pennies, 15 nickels, and 15 quarters.

Section 3: Activity 2

1. $\dfrac{\text{amount of flour (mL)}}{\text{amount of sugar (mL)}} \quad \dfrac{2}{1} \overset{\times 175}{=} \dfrac{350}{175}$

Tom needs 175 mL of sugar.

2. $\dfrac{\text{number of cooks}}{\text{number of waiters and waitresses}} \quad \dfrac{3}{5} \overset{\times 3}{=} \dfrac{9}{15}$

There are 15 waiters and waitresses.

3. $\frac{\text{number of canoes}}{\text{number of sailboats}}$

$\frac{7}{2} \overset{\times 3}{=} \frac{21}{6}$

There are 6 sailboats.

4. $\frac{\text{number of bones in the head}}{\text{total number of bones}}$

$\frac{1}{7} \overset{\times 29}{=} \frac{29}{206}$

There are about 29 bones in the head.

5. $\frac{\text{mass of silver (g)}}{\text{total mass (g)}}$

$\frac{37}{40} \overset{\times 10}{=} \frac{370}{400}$

There are 370 g of silver in the cup.

6. $\frac{\text{distance (m)}}{\text{time (s)}}$

$\frac{2.5}{1} \overset{\times 50}{=} \frac{125}{50}$

At this rate, Arlene can swim 125 m.

7. **Step 1:** Simplify the rate.

$$\frac{450}{40} = \frac{45}{4}$$

Step 2: Write a proportion and find the missing term.

$\frac{\text{earnings (\$)}}{\text{time (h)}}$

$\frac{45}{4} \overset{\times 9}{=} \frac{405}{36}$

At this rate, René will earn \$405.

8. **Step 1:** Simplify the rate.

$$\frac{25}{2.5} = \frac{10}{1}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$\frac{\text{distance (km)}}{\text{time (h)}}$

$\frac{10}{1} \overset{\times 5}{=} \frac{50}{5}$

At this rate, Chester will need 5 h to travel 50 km.

9. Step 1: Simplify the rate.

$$\frac{1000}{15} = \frac{200}{3}$$

Step 2: To solve the problem, write a proportion and find the missing term.

$$\frac{\text{amount (Japanese yen)}}{\text{amount (Canadian dollars)}} \qquad \frac{200}{3} \overset{\times 25}{=} \frac{5000}{75}$$

You can buy 5000 Japanese yen for $75 Canadian.

Now Try This

10. Step 1: Find Geoff's race time. **Hint:** If Geoff gets a 10-m start in the 100-m race, he has only 90 m left to run.

$$\frac{\text{distance (m)}}{\text{time (s)}} \qquad \frac{5}{1} \overset{\times 18}{=} \frac{90}{18}$$

With a 10-m head start, Geoff would run the 100-m race in 18 s.

Step 2: Find Simon's race time.

$$\frac{\text{distance (m)}}{\text{time (s)}} \qquad \frac{7.5}{1} = \frac{100}{13.3}$$

Simon would run the race in 13.3 s.

Step 3: Compare the race times.

$$13.3 < 18$$

So, Geoff would not win with a 10-m head start.

11. Step 1: Change the 200-m time to seconds.

$$1 \text{ min } 52.8 \text{ s} = 112.8 \text{ s}$$

Suki swims the 200-m race in 112.8 s.

Step 2: Find the time for a 200-m race at the 100-m pace. Write a proportion and find the missing term.

$$\frac{\text{distance (m)}}{\text{time (s)}} \qquad \frac{100}{51.2} \overset{\times 2}{=} \frac{200}{102.4}$$

At this rate, Suki would swim the 200-m race in 102.4 s.

Step 3: Find the difference in the times.

$$\begin{array}{r} 112.8 \\ -\ 102.4 \\ \hline 10.4 \end{array}$$

Suki would cut 10.4 s off her record.

12. Step 1: Calculate each rate.

	Nestor	Leland
$\frac{\text{number of words}}{\text{time (min)}}$	$\frac{30}{1}$	$\frac{20}{1}$

Step 2: Calculate the time for Nestor to type 60 words.

$\frac{\text{number of words}}{\text{time (min)}}$ $\quad \frac{30}{1} \overset{\times 2}{=} \frac{60}{2}$

It will take Nestor 2 min to type 60 words.

Step 3: Calculate the time for Leland to type 60 words.

$\frac{\text{number of words}}{\text{time (min)}}$ $\quad \frac{20}{1} \overset{\times 3}{=} \frac{60}{3}$

It will take Leland 3 minutes to type 60 words.

Step 4: Calculate the difference in time.

$$3 - 2 = 1$$

It will take Leland 1 min longer to type 60 words.

13. If the number has no remainder when divided by 7, it is a multiple of 7.

The multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56,… .

Use the guess, check, and revise strategy to find the number.

Guess 1: The number is 7.

$7 \div 2 = 3$ R1

$7 \div 3 = 2$ R1

$7 \div 4 = 1$ R3 ← This does not meet the condition.

Guess 2: The number is 14.

$14 \div 2 = 7$ ← This does not meet the condition.

Guess 3: The number is 21.

$21 \div 2 = 10$ R1

$21 \div 3 = 7$ ← This does not meet the condition.

Guess 4: The number is 28.

$28 \div 2 = 14$ ← This does not meet the condition.

Guess 5: The number is 35.

$35 \div 2 = 17$ R1

$35 \div 3 = 11$ R2 ← This does not meet the condition.

Guess 6: The number is 42.

$42 \div 2 = 21$ ← This does not meet the condition.

Guess 7: The number is 49.

$49 \div 2 = 24$ R1

$49 \div 3 = 16$ R1

$49 \div 4 = 22$ R1

$49 \div 6 = 8$ R1

$49 \div 7 = 7$ R0 ← This meets the condition.

All conditions are met. So, the number is 49.

Section 3: Activity 3

1. Step 1: Write the percent as a ratio in simplest form.

$$23\% = \frac{23}{100}$$

Step 2: Write a proportion and find the missing term.

$$\frac{\text{amount of milk fat (g)}}{\text{total mass (g)}} \qquad \frac{23}{100} \overset{\times 2}{=} \frac{46}{200}$$

About 46 g of milk fat are in the cheese.

2. Step 1: Write the percent as a ratio in simplest form.

$$15\% = \frac{15}{100}$$

$$= \frac{3}{20}$$

Step 2: Write a proportion and find the missing term.

$$\frac{\text{amount of tip (\$)}}{\text{amount of bill (\$)}} \qquad \frac{3}{20} \overset{\times 3}{=} \frac{9}{60}$$

The Allans' food bill was \$60.

3. Step 1: Write the percent as a ratio in simplest form.

$$130\% = \frac{130}{100}$$
$$= \frac{13}{10}$$

Step 2: Write a proportion and find the missing term.

$$\frac{\text{profits this year (\$)}}{\text{profits last year (\$)}} \qquad \frac{13}{10} \overset{\times 520}{=} \frac{6760}{5200}$$

The profits this year were \$6760.

4. Step 1: Write the percent as a ratio in simplest form.

$$80\% = \frac{80}{100}$$
$$= \frac{4}{5}$$

Step 2: Write a proportion and find the missing term.

$$\frac{\text{number of students with part-time jobs}}{\text{total number of students}} \qquad \frac{4}{5} \overset{\times 30}{=} \frac{120}{150}$$

There are 150 students in the school.

Now Try This

You can use logic to solve problems 5 to 7.

5. Step 1: Find the percent of cattle sold to the fourth buyer.

$$50\% + 25\% + 20\% = 95\%$$

So, 95% of the cattle were sold to the first, second, and third buyers.

$$100\% - 95\% = 5\%$$

So, 5% of the cattle were sold to the fourth buyer.

Step 2: Write the percent as a ratio in simplest form.

$$5\% = \frac{5}{100}$$
$$= \frac{1}{20}$$

Step 3: To solve the problem, write a proportion and find the missing term.

$$\frac{\text{number of cattle sold to the fourth buyer}}{\text{total number of cattle}} \qquad \frac{1}{20} \overset{\times 7}{=} \frac{7}{140}$$

The farmer had 140 cattle.

6. **Step 1:** Write the percent as a ratio in simplest form.

$$80\% = \frac{80}{100}$$
$$= \frac{4}{5}$$

Step 2: Find the ratio of the correct responses to the number of incorrect responses. Use this reasoning.

If the ratio of the number of correct responses to the total number of responses is **4** to 5, the ratio of the number of incorrect responses to the total number of responses is **1** to 5.

So, the ratio of the number of correct responses to the number of incorrect responses is 4 to 1.

7. **a.** **Step 1:** Find the ratio of the amount of tin to the amount of bronze. Use this reasoning.

If the ratio of the amount of copper to the amount of tin is 9 to 1, the ratio of the amount of tin to the amount of bronze is 1 to 10.

Step 2: Find the percent of tin in the bronze frame.

$$\frac{\text{amount of tin}}{\text{amount of bronze}} \qquad \frac{1}{10} = \frac{10}{100}$$
$$= 10\%$$

So, 10% of the frame is tin.

b. $100\% - 10\% = 90\%$

So, 90% of the frame is copper.

8. You may use guess, check, and revise strategy to solve this problem.

Step 1: Find the number of free throws needed to make 50%.

Number of Consecutive Free Throws Made	Total Number of Attempts
9	26
10	27
11	28
12	29
13	30
14	31
15	32
16	33
17	34 ← Correct guess

$$\frac{17}{34} = \frac{1}{2}$$
$$= 50\%$$

So, Ernie will have to make 17 out of 34 free throws to raise his record to 50%.

Step 2: Find the number of free throws required. Ernie has already made 8 good throws.

$$17 - 8 = 9$$

So, Ernie will need to make the next 9 free throws.

Section 3: Follow-up Activities

Extra Help

1. Step 1: Write a proportion.

number of passes completed to number of passes attempted

$$2:5 = 12:\square$$

Step 2: Find the missing term.

$$2:5 \xrightarrow{\times 6,\ \times 6} 12:30$$

Denzil attempted 30 passes.

2. a. Step 1: Write a proportion.

number of revolutions of the crank gear to number of revolutions of the wheel

$$7:10 = \square:120$$

Step 2: Find the missing term.

$$7:10 \xrightarrow{\times 12,\ \times 12} 84:120$$

So, 84 crank gear revolutions are needed in first gear.

b. Step 1: Write a proportion.

number of revolutions of the wheel to number of revolutions of the crank gear

$$1:3 = \square:120$$

Step 2: Find the missing term.

$$1:3 \xrightarrow{\times 40,\ \times 40} 40:120$$

So, 40 crank gear revolutions are needed in tenth gear.

3. **Step 1:** Write a proportion.

number of blinks per minute

$$25:1 = \square:60$$

Step 2: Find the missing term.

$$25:1 \quad (\times 60,\ \times 60) \quad = 1500:60$$

You will blink 1500 times in an hour.

4. **Step 1:** Write a proportion.

number of characters per minute

$$900:1 = 7200:\square$$

Step 2: Find the missing term.

$$900:1 \quad (\times 8,\ \times 8) \quad = 7200:8$$

It will take 8 min to print the characters.

5. **Step 1:** The percent cannot be simplified. So, write a proportion.

number of children born outside Canada to total number of children

$$9:100 = \square:200$$

Step 2: Find the missing term.

$$9:100 \quad (\times 2,\ \times 2) \quad = 18:200$$

So, 18 of the children were born outside Canada.

6. **Step 1:** Write the percent as a ratio in simplest form.

$$25\% = \frac{25}{100}$$
$$= \frac{1}{4}$$
$$= 1:4$$

Step 2: Write a proportion.

number of incisors to total number of teeth

$$1:4$$
$$= \square : 32$$

Step 3: Find the missing term.

$$1:4$$
$$\times 8 \quad \times 8$$
$$= 8:32$$

There are 8 incisors.

7. **Step 1:** Write the percent as a ratio in simplest form.

$$300\% = \frac{300}{100}$$
$$= \frac{3}{1}$$
$$= 3:1$$

Step 2: Write a proportion.

number of wins to number of losses

$$3:1$$
$$= \square : 4$$

Step 3: Find the missing term.

$$3:1$$
$$\times 4 \quad \times 4$$
$$= 12:4$$

The Flyers won 12 games.

Enrichment

1.

Type of Dairy Product	Milk Fat (%)	Amount of Milk Fat in 1000 mL
whipping cream	35	350
coffee cream	18	180
cereal cream	9	90
partly skimmed milk	1	10

2. $65\%\ \text{of}\ 5 = 0.65 \times 5$
$= 3.25$

There are 3.25 kg of tin.

3. $75\%\ \text{of}\ 10\,800 = 0.75 \times 10\,800$
$= 8100$

There were 8100 people at the football game.

4. 1 6 0 0 × 1 5 %

240.

They will travel 240 km before lunch.

5. 1 2 0 × 3 5 %

42.

The librarian bought 42 adventure stories.

6. 1 5 × 4 0 0 %

60.

There are 60 red balloons.

7. 3 8 0 × 1 2 0 %

456.

Mrs. Ricci spends $456.

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